An ergodic BSDE approach to forward entropic risk measures: representation and large-maturity behavior

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Joint work with Ying Hu (Université de Rennes 1), Gechun Liang (King's College London), Thaleia Zariphopoulou (The University of Texas at Austin)

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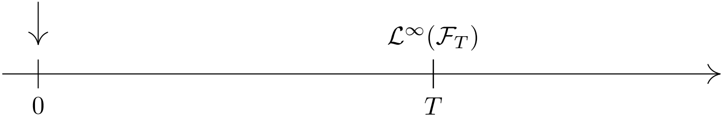

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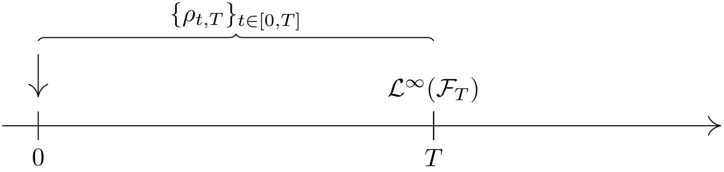

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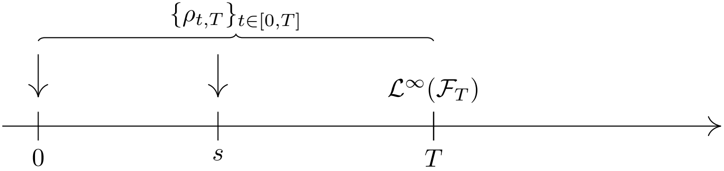

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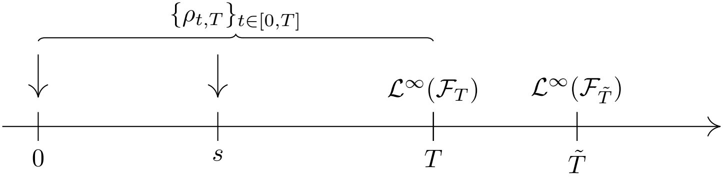

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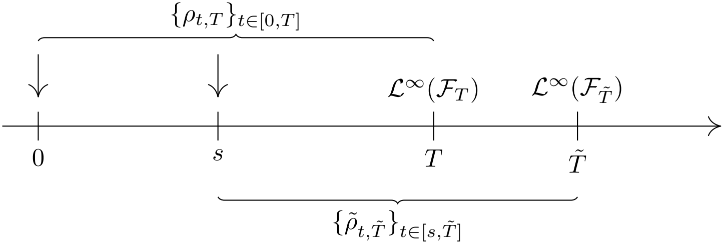

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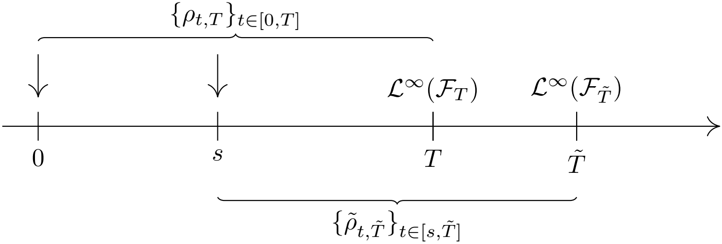

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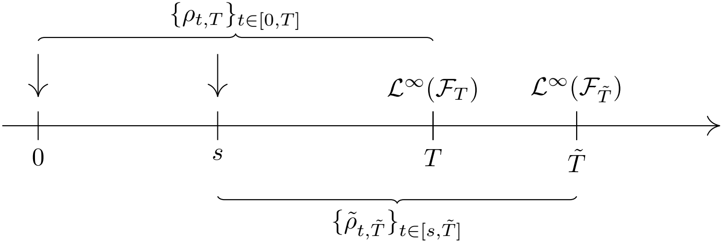

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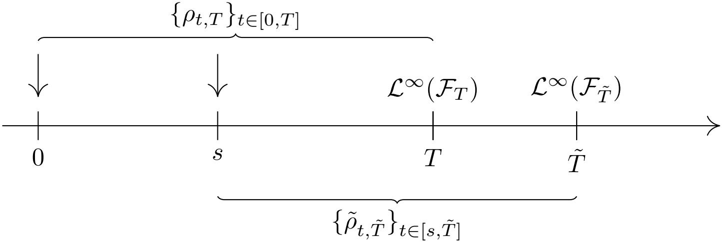

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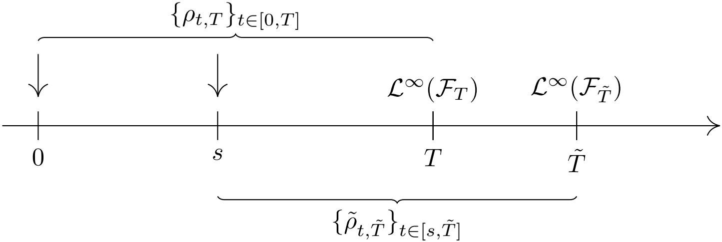

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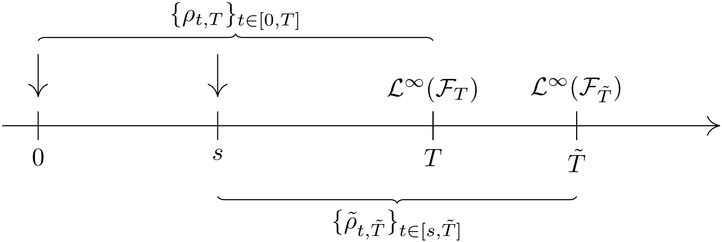

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Maturity independent risk measure by Zariphopoulou and Žitković (2010)

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Let $T \geq 0$ be fixed, and consider a risk position with payoff $\xi_T \in \mathcal{L}^\infty(\mathcal{F}_T)$

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How could we know our utility at time T?

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Musiela and Zariphopoulou (2005, 2007, 2008, 2009a,b, 2010a,b)

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Moreover, a forward investment performance process $\{U(\omega, x, t)\}_{\omega\in\Omega, x\in\mathbb{R}, t\geq 0}$ is called an exponential forward investment performance process, if there exist a constant $\gamma > 0$ and an $\mathbb{F}$-progressively measurable process $K = \{K_t\}_{t\geq 0}$ such that

$$U(x, t) = -e^{-\gamma x + K_t}, \quad \forall (x, t) \in \mathbb{R} \times [0, T].$$

Stochastic Factor Market Model

Let $W = \{W_t\}_{t \geq 0}$ be a d-dimensional Brownian motion on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F} = \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$ with the augmented filtration $\mathbb{F}$ generated by W.

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The market consists of $n(\leq d)$ (discounted) risky stocks $S = \{(S_t^1, S_t^2, \dots, S_t^n)^{tr}\}_{t\geq 0}$, with the dynamics, for $i = 1, \dots, n$,

$$\frac{dS_t^i}{S_t^i} = b^i(V_t)dt + \sigma^i(V_t)dW_t, \quad \forall t \in [0, \infty).$$

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Assume that (A1) $b^i(v) \in \mathbb{R}$ and $\sigma^i(v) \in \mathbb{R}^{1\times d}$ are uniformly bounded in $v \in \mathbb{R}^d$; (A2) $\sigma(v) = (\sigma^1(v), \ldots, \sigma^n(v))^{tr}$ has full row rank n, and hence the market price of risk

$$\theta(v) = \sigma(v)^{tr}[\sigma(v)\sigma(v)^{tr}]^{-1}b(v), \quad \forall v \in \mathbb{R}^d,$$

exists, where $b(v) = (b^1(v), \ldots, b^n(v))^{tr}$; (A3) $\theta(v) \in \mathbb{R}^d$ is uniformly bounded and Lipschitz continuous in $v \in \mathbb{R}^d$.

Stochastic Factor Market Model (cont.)

$V = \{(V_t^1, V_t^2, \ldots, V_t^d)^{tr}\}_{t \geq 0}$ is the d-dimensional stochastic factor, with the dynamics,

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Assume that (A4) $\eta(v) \in \mathbb{R}^d$ satisfies the dissipative condition that there exists a constant $C_\eta (> C_v > 0)$ such that

$$(\eta(v) - \eta(\bar{v}))^{tr} (v - \bar{v}) \leq -C_\eta |v - \bar{v}|^2, \quad \forall v, \bar{v} \in \mathbb{R}^d;$$

(A5) $\kappa \in \mathbb{R}^{d \times d}$ is positive definite and normalized to $|\kappa| = 1$.

Stochastic Factor Market Model (cont.)

Denote $\tilde{\pi} = \{(\tilde{\pi}_t^1, \tilde{\pi}_t^2, \ldots, \tilde{\pi}_t^n)^{tr}\}_{t \geq 0}$ as the amount investing to the risky assets S

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$$dX_t^\pi = \pi_t^{tr}(\theta(V_t)dt + dW_t), \quad \forall t \in [0, \infty),$$

where $\pi_t^{tr} = \tilde{\pi}_t^{tr}\sigma(V_t)$

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$$\mathcal{A}[0,t] = \{\pi = \{\pi_s\}_{s\in[0,t]} : \pi \in \mathcal{L}_{BMO}^2[0,t] \text{ and } \pi_s \in \Pi, \forall s \in [0,t]\},$$

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where

$$\mathcal{L}^2_{BMO}[0,t] = \Bigg\{\pi = \{\pi_s\}_{s\in[0,t]} : \pi \text{ is } \mathbb{F}\text{-progessively measureable and } \operatorname*{ess\,sup}_{\tau\in\mathcal{T}[0,t]} \mathbb{E}\left[\int_\tau^t |\pi_s|^2 ds \middle| \mathcal{F}_\tau\right] < \infty\Bigg\},$$

$\mathcal{T}[0,t]$ is the set of $\mathbb{F}$-stopping time $\tau \in [0,t]$, and Π is a closed and convex subset of $\mathbb{R}^d$

Stochastic Factor Market Model (cont.)

Denote $\tilde{\pi} = \{(\tilde{\pi}_t^1, \tilde{\pi}_t^2, \ldots, \tilde{\pi}_t^n)^{tr}\}_{t\geq 0}$ as the amount investing to the risky assets S. The wealth process $X^\pi = \{X_t^\pi\}_{t\geq 0}$ satisfies

$$dX_t^\pi = \pi_t^{tr}(\theta(V_t)dt + dW_t), \quad \forall t \in [0, \infty),$$

where $\pi_t^{tr} = \tilde{\pi}_t^{tr}\sigma(V_t)$. For each $t \geq 0$, denote $\mathcal{A}[0,t]$ as the set of admissible investment strategies defined for times $s \in [0,t]$:

$$\mathcal{A}[0,t] = \{\pi = \{\pi_s\}_{s\in[0,t]} : \pi \in \mathcal{L}^2_{BMO}[0,t] \text{ and } \pi_s \in \Pi, \forall s \in [0,t]\},$$

where

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$\mathcal{T}[0,t]$ is the set of $\mathbb{F}$-stopping time $\tau \in [0,t]$, and Π is a closed and convex subset of $\mathbb{R}^d$. Then, the set $\mathcal{A}$ of admissible investment strategies defined for times $t \geq 0$ is defined by

$$\mathcal{A} = \cup_{t\geq 0}\mathcal{A}[0,t].$$

Ergodic BSDE Representations for Exponential Forward Investment Performance Processes

Liang and Zariphopoulou (2017)

Define

$$U(x,t) = -e^{-\gamma x + Y_t - \lambda t}, \quad \forall (x,t) \in \mathbb{R} \times [0,\infty), \tag{1}$$

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where $(Y = \{Y_t\}_{t\geq 0}, Z = \{Z_t\}_{t\geq 0}, \lambda)$ is a solution, if exist, of the ergodic (quadratic) BSDE

$$dY_t = (-F(V_t, Z_t) + \lambda)dt + Z_t^{tr} dW_t, \quad \forall t \in [0,\infty), \tag{2}$$

with the driver $F : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}$ defined by

$$F(v,z) = \frac{\gamma^2}{2}\text{dist}^2\left\{\Pi, \frac{z+\theta(v)}{\gamma}\right\} - \frac{1}{2}|z+\theta(v)|^2 + \frac{1}{2}|z|^2, \quad \forall v, z \in \mathbb{R}^d. \tag{3}$$

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Suppose that (A1) – (A5) hold. Liang and Zariphopoulou (2017) showed

(i) the ergodic BSDE (2) admits a unique Markovian solution with functions y (unique up to an additive constant, at most linear growth) and z (uniformly bounded) with respect to the stochastic factor; and

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(ii) $U(x,t)$ in (1) is an exponential forward investment performance process.

Main Result 1: BSDE Representations for Forward Entropic Risk Measures

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Let $T \geq 0$ be arbitrary, and consider a risk position with payoff $\xi_T \in \mathcal{L}^\infty(\mathcal{F}_T)$

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$$Y_t^{\xi_T} = \xi_T + \int_t^T G(V_u, Z_u, Z_u^{\xi_T}) du - \int_t^T (Z_u^{\xi_T})^{tr} dW_u, \tag{4}$$

where the driver $G : \mathbb{R}^d \times \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}$ is defined as

$$G(v, z, \bar{z}) = \frac{1}{\gamma} \left(F(v, z + \gamma \bar{z}) - F(v, z) \right), \quad \forall v, z, \bar{z} \in \mathbb{R}^d, \tag{5}$$

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(i) The BSDE (4) has a unique solution $(Y^{\xi_T} = \{Y_t^{\xi_T}\}_{t \in [0,T]}, Z^{\xi_T} = \{Z_t^{\xi_T}\}_{t \in [0,T]})$ with Y^{ξ_T} being uniformly bounded and $Z^{\xi_T} \in \mathcal{L}^2_{BMO}[0, T]$;

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(ii) The forward entropic risk measure of ξ_T is given, for $t \in [0,T]$, by

$$\rho_t(\xi_T) = Y_t^{-\xi_T}.$$

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$$G^*(v, z, q) = \sup_{\bar{z} \in \mathbb{R}^d} \left(\bar{z}^{tr} q - G(v, z, \bar{z}) \right), \quad \forall q \in \mathbb{R}^d.$$

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Define the admissible set

$$\mathcal{A}^*[0, T] = \left\{ q \in \mathcal{L}^2_{BMO}[0, T] : \mathbb{E}^{\mathbb{Q}^q} \left[\int_0^T |G^*(V_s, Z_s, q_s)| ds \right] < \infty \right\},$$

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where, for any $q \in \mathcal{L}^2_{BMO}[0, T]$, the probability measure $\mathbb{Q}^q$ on $\mathcal{F}_T$ is defined by

$$\frac{d\mathbb{Q}^q}{d\mathbb{P}} = \mathcal{E} \left(\int_0^\cdot q_s^{tr} dW_s \right)_T.$$

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(i) The forward entropic risk measure $\rho_t(\xi_T)$ admits the following convex dual representation:

$$\rho_t(\xi_T) = -\operatorname*{ess\,inf}_{q \in \mathcal{A}^*[t,T]} \mathbb{E}^{\mathbb{Q}^q}\left[\xi_T + \int_t^T G^*(V_s, Z_s, q_s) ds \middle| \mathcal{F}_t \right].$$

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(ii) There exists an optimal $q^* \in \mathcal{A}^*[t,T]$ such that

$$\rho_t(\xi_T) = -\mathbb{E}^{\mathbb{Q}^{q^*}} \left[\xi_T + \int_t^T G^*(V_s, Z_s, q_s^*) ds \middle| \mathcal{F}_t \right].$$

Corollary of Main Results 1 and 2: Maturity Independent Convex Risk Measure

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(i) Cash-invariance: $\rho_t(\xi_T - m) = \rho_t(\xi_T) + m$, for any $m \in \mathcal{L}^\infty(\mathcal{F}_t)$ and $\xi_T \in \mathcal{L}^\infty(\mathcal{F}_T)$;

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(iii) Convexity: $\rho_t(\alpha\xi_T + (1-\alpha)\bar{\xi}_T) \leq \alpha\rho_t(\xi_T) + (1-\alpha)\rho_t(\bar{\xi}_T)$, for any $\alpha \in [0,1]$ and $\xi_T, \bar{\xi}_T \in \mathcal{L}^\infty(\mathcal{F}_T)$;

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(iv) Time-consistency: $\rho_t(\xi_T) = \rho_t(-\rho_s(\xi_T))$, for any $0 \leq t \leq s \leq T < \infty$.

Main Result 3: Large Maturity Behavior for Forward Entropic Risk Measures

Chong, Hu, Liang, and Zariphopoulou (2017)

Let $T \geq 0$ be arbitrary, and consider a risk position with payoff

$$\xi_T = -g\left(V_T\right),$$

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$$\lim_{T\uparrow\infty} \rho_0(\xi_T) = \lim_{T\uparrow\infty} y^{T,g}(v,0) = L^g.$$

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$$\lim_{T\uparrow\infty} \rho_0(\xi_T) = \lim_{T\uparrow\infty} y^{T,g}(v,0) = L^g.$$

Moreover, for any $T > 0$,

$$|\rho_0(\xi_T) - L^g| \leq C(1 + |v|^2)e^{-\hat{C}_\eta T},$$

with some constants $C, \hat{C}_\eta$.

Example: One Stock, One Stochastic Factor, and Two Brownian Motions

Consider

$$dS_t = b\left(V_t\right)S_t dt + \sigma\left(V_t\right)S_t dW_t^1, \quad \forall t \in [0, \infty),$$

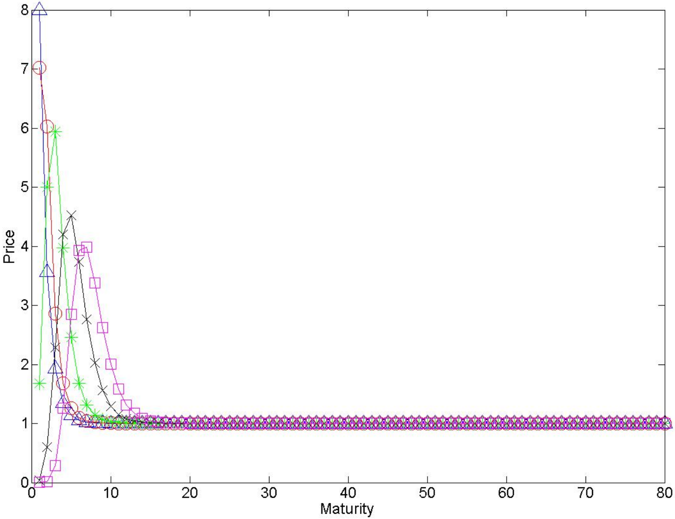
$$dV_t = \eta\left(V_t\right)dt + \kappa^1 dW_t^1 + \kappa^2 dW_t^2, \quad \forall t \in [0, \infty).$$

Let $\Pi = \mathbb{R} \times \{0\}$. Then the forward entropic risk measure has the closed-form representation

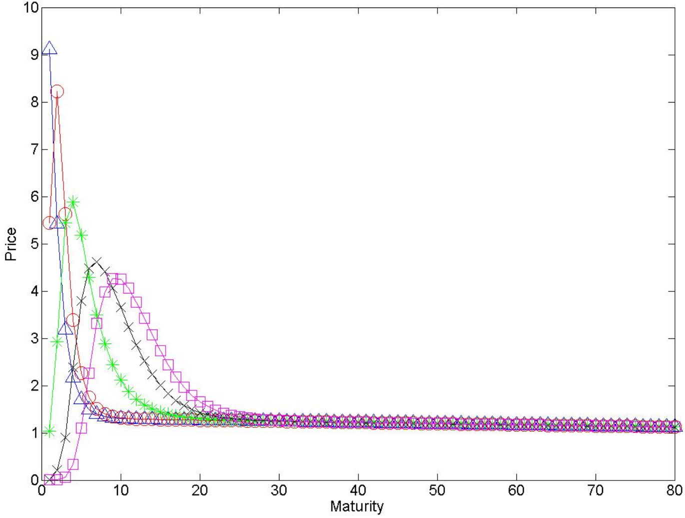
$$\rho_t(\xi_T) = Y_t^{-\xi_T} = \frac{1}{\gamma|\kappa_2|^2} \ln \mathbb{E}^{\mathbb{Q}}\left[e^{\gamma|\kappa_2|^2 g(V_T)}|\mathcal{F}_t\right].$$

Assume that $\gamma = 1$, $\theta(v) = (K_1 - |v|)_+$ with $K_1 = 10$, $\eta(v) = -0.1v$, $g(v) = (K_2 - |v|)_+$ with $K_2 = 10$.

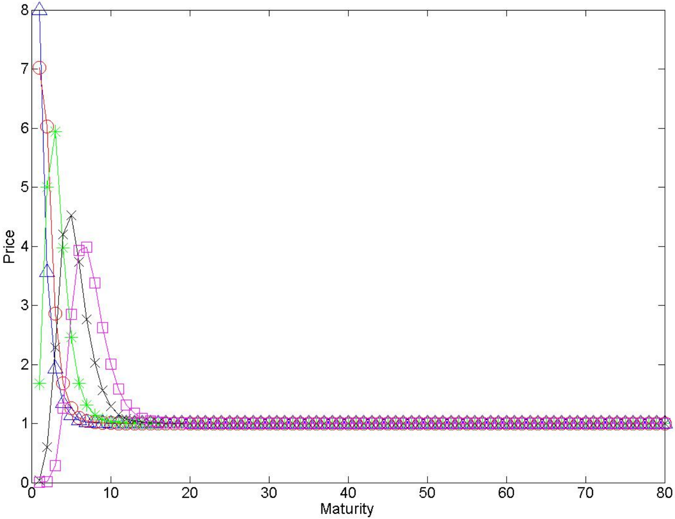
Example: One Stock, One Stochastic Factor, and Two Brownian Motions (cont.) $\kappa_1 = 0.9$, $\kappa_2 = 0.1$



Example: One Stock, One Stochastic Factor, and Two Brownian Motions (cont.) $\kappa_1 = 0.5$, $\kappa_2 = 0.5$



Example: One Stock, One Stochastic Factor, and Two Brownian Motions (cont.) $\kappa_1 = 0.0$, $\kappa_2 = 1.0$



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Thank you!