Exhaustible Resources with Exploration and Production Adjustment Costs

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Outline

Introduction: Some Stylized Facts in Real Markets I

- "There are only a few fields in economics whose antecedents can be traced to a single, seminal article. One such field is natural resource economics, which is currently experiencing an explosive revival of interest; its origin is widely recognized as Harold Hotelling's 1931 paper, The Economics of Exhaustible Resources." S. Devarajan and A.C. Fisher, JEL (1981)
- By sovling a deterministic control problem, the model of Hotelling (1931) implies that the price of oil, as an exhaustible resource, will go up exponentially in equilibrium.
- However, historically the oil prices can have complicated dynamics, rather than going up exponetially.
- For example, the recent low oil price around $50 per barrel since January 2015 has been drawn media attentions. Historically, there are periods during which prices can be high or low. (Source: www.macrotrends.net)

Introduction: Some Stylized Facts in Real Markets II

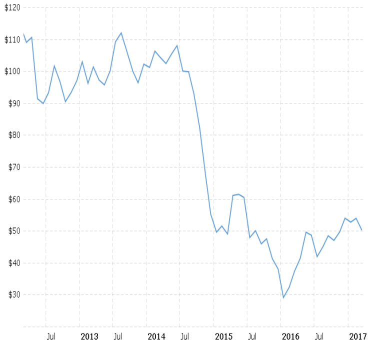

Figure: Oil price for the last 5 years

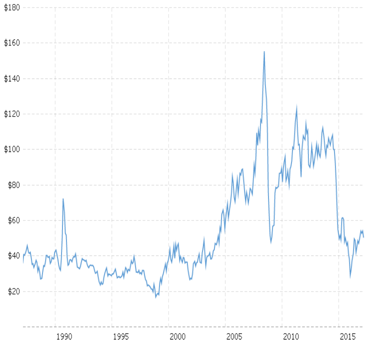

Figure: Oil price for the last 30 years

Introduction: Some Stylized Facts in Real Markets III

- The term structure of futures curves exhibit both backwardation and contango. (*Litzenberger and Robinowitz 1995, JF*)

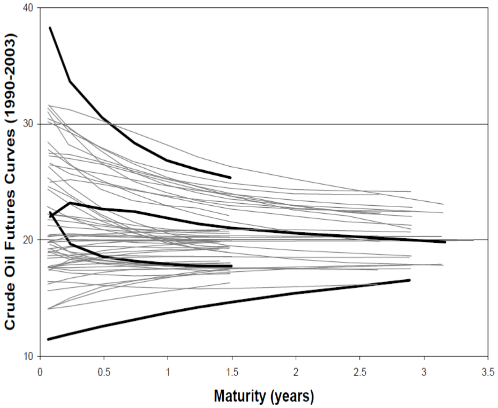

Figure: Backwardation and contango. (from *Casassus et al. 2005*)

Introduction: Some Stylized Facts in Real Markets IV

- (1) Samuelson effect (*Samuelson 1965*): Term structure curve is downward sloping; (2) volatility of futures prices is higher when the futures curve is in backwardation. (*Routledege et al 2000, JF*)

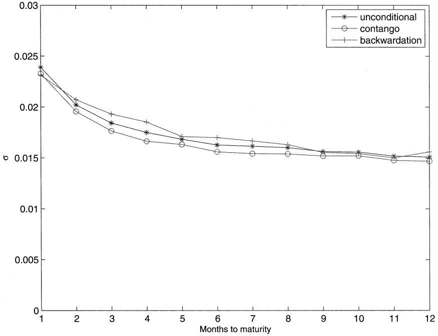

Figure: Term structure of futures volatility. (from *Kogan et al. 2009, JF*)

A Short Summary

From the above stylized facts, we can conclude:

- prices can be volatile (stochastic);
- oil industry has its own production logistics, which leads to complicated price dynamics;
- prices can stay at a relative stable level for a quite long time.

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This paper is devoted to propose a general equilibrium model to provide possible explanations to all these stylized facts by incoporating

- stochastic demand;
- production adjustment costs;
- exploration.

Exploration

In reality, the ultimate availability of the resource depends upon the outcome of exploration activities. In fact, for many exhaustible resources, reserves have actually increased for over the last 100 years.

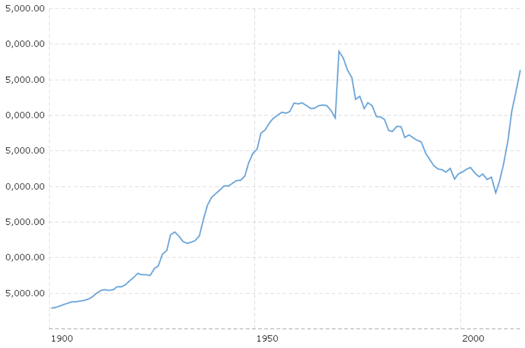

Figure: The world oil reserves. (Source: www.macrotrends.net)

Production Adjustment Costs

Since resource production is a capital intensive activity, adjustment costs cannot be avoided during the production. Indeed, production adjustment costs can occured in all levels.

- Unit level: direct adjustment costs. For example, Canada Oil Sands–Turning on and off bitumen production is a complex and lengthy process. Stopping the injection of steam into oil sand reservoirs would result in a long and expensive re-start.
- Firm level: set-up costs, hiring costs, firing costs, etc.
- Industry level: entry and exit costs.

Exploration and Production Adjustment Costs

When taking into account both exploration and adjustment costs, two questions naturally arized:

- Theoretically, how do they affect the equilibrium of exhaustible resources?
- Numerically, can they help to explain the above mentioned stylized facts?

Method Used: Singular Stochastic Control

By applying singular stochastic control, we can explicitly model upward and downward adjustment costs which is proportional to the magnitude of adjustments.
Why use singular control?

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Why use singular control?

- Singular control is a natural way to introduce adjustment costs without introducing any other adjustment factors, as upward and downward adjustment do occur in practice.
- From a technical point, the model is still analytically tractable, as semi-closed form solutions are available.
- More generally, the initial upward adjustment cost is a parsimonious way to model the set-up cost, and the upward and downward adjustment costs can also be viewed as entry and exit costs respectively.

Our main results are as follows:

- First, theoretically, we show that there exists a unique extraction path of an exhaustible resource with production adjustment costs in the socially optimal way, which can be reproduced by a competitive market equilibrium.
- Second, adjustment costs combined with uncertainties (e.g., demand and/or reserve shocks) can generate many interesting phenomena such as (a) backwardation and contango, (b) Samuelson effect, and (c) higher volatility conditional on backwardation.
- Finally, compared to either using exploration or production adjustments only, the combination of both exploration and production adjustments costs can significantly prolong the price staying at the bottom.

Related Literature

- **Production adjustment costs**. There are very few works in the literature.
  Carlson, Khokher, and Titman, 2007, JF, consider a model that adjustment costs are incurred per unit time when production rate exceeds its historic weighted average.
  This is similar to the average capital gain tax in the transaction cost literture
  Some limitations of the model:
  - Several production adjustment costs are not included, e.g. set-up costs.
  - The weighted historic average is hard to compute analytically.

  Our model considers instant upward and downward production costs, similar to the transaction costs of purchasing stocks.

Related Literature (Cont'd)

- **Exploration**. There are many studies focusing on exploration activities but no works consider exploration and adjustment costs simultaneously. For example,
  Pindyck 1978, JPE (deterministic)
  Pindyck 1980, JPE (stochastic)
  Arrow and Chang, 1982, JEEM (jump in exploration)
- In particular, *Pindyck 1978, JPE and Pindyck 1980, JPE* are our special cases in the absence of adjustment costs.

Related Literature (Cont'd)

Table: A Short Summary of Literature

Model	Demand Uncertainty	Exploration	Adjustment Costs
Pindyck (1978, JPE)		✓	
Pindyck (1980, JPE)	✓	✓	
Arrow & Chang (1982)		✓	
Carlson et al (2007, JF)	✓		✓
This Paper	✓	✓	✓

Basic Model: Adjustment Costs in Deterministic Case

- The inverse demand function is of a general form:

$$p = P(Q) \quad \text{with} \quad P'(Q) < 0,$$

  where $Q \geqslant 0$ is the aggregate production rate.
- Denote by I_t and D_t the cumulative upward and downward adjustment of production rate up to time t, respectively. Then the production rate is

$$dQ_t = dI_t - dD_t, \qquad Q_{0^-} = q. \tag{1}$$

  Consequently, the reserve R is

$$dR_t = -Q_t dt, \qquad R_0 = r. \tag{2}$$

- The adjustment costs at time t can be expressed as follows.

$$\eta_+ dI_t + \eta_- dD_t,$$

  where $\eta_\pm$ are respectively marginal proportional costs of upward and downward adjustments, satisfying $\eta_+ > 0$ and $\eta_- + \eta_+ \geqslant 0$.
- No storage and constant marginal extraction cost C.

Basic Model: A Social Planner's Problem

As in the classical Hotelling model, the social objective function is the discounted sum of social surplus nets of costs:

$$S(\{I_t, D_t\}) := \int_0^\infty e^{-\beta t}\Big([U(Q_t) - C\,Q_t]dt - \eta_+ dI_t - \eta_- dD_t\Big),$$

where $U(\cdot)$ is the social utility function with the form: $U(q) = \int_0^q P(x)dx$.

Now, for a given initial state $(r,q) \in [0,\infty)^2$, the social planner's problem can be described as the following singular control problem:

$$V(r,q) := \sup_{\{I_t, D_t\}} S(\{I_t, D_t\}),$$

subject to the dynamics (1) and (2). The corresponding HJB equation:

$$\begin{cases} \max\left\{\mathcal{L}V + U(q) - C\,q,\ \dfrac{\partial V}{\partial q} - \eta_+,\ -\dfrac{\partial V}{\partial q} - \eta_-\right\} = 0, & \text{for } r > 0,\ q \geqslant 0, \\ V(0,q) = -\eta_- q, & \text{for } q \geqslant 0. \end{cases}$$

where $\mathcal{L}V = -q\frac{\partial V}{\partial r} - \beta V$.

No Adjustment Costs

The classical Hotelling model (no adjustment costs) is our special case with $\eta_{\pm} = 0$, then

$$V^H(r) = \sup_{\{Q \geqslant 0\}} \int_0^{\infty} e^{-\beta t}[U(Q_t) - C\, Q_t]dt,$$

and the corresponding HJB equation:

$$\max_{q \geqslant 0} \left\{ U(q) - C\, q - q\frac{dV^H}{dr} \right\} - \beta V^H = 0.$$

The first order condition implies

$$\frac{dV^H}{dr} = \frac{dU}{dq} - C = P - C. \tag{3}$$

It states that the marginal value of a resource must equal the marginal utility minus marginal extraction costs. Alternatively, *scarcity rent* equals *net price*.

In a competitive equilibrium:

$$\frac{d}{dt}[P(Q_t) - C] = \beta[P(Q_t) - C] \quad \Longleftrightarrow \quad \frac{d\lambda_t^H}{dt} = \beta \lambda_t^H,$$

where $\lambda_t^H := \frac{dV^H}{dr}|_{r=R_t}$ is the scarcity rent or shadow price.

Optimal Policy with Adjustment Costs

Motived by the HJB equation, we can define the following three regions:

- **Upward-adjustment region**:

$$\mathcal{I} := \left\{ (r, q) \in [0, \infty)^2 \;\middle|\; \frac{\partial V}{\partial q} = \eta_+ \right\}$$

  states that the marginal value of an upward adjustment must equal its marginal cost.

- **Downward-adjustment region**:

$$\mathcal{D} := \left\{ (r, q) \in [0, \infty)^2 \;\middle|\; -\frac{\partial V}{\partial q} = \eta_- \right\}$$

  states that the marginal value of a downward adjustment must equal its marginal cost.

Optimal Policy with Adjustment Costs

- **No-adjustment region**:

$$\mathcal{N} := \left\{(r,q) \in [0,\infty)^2 \;\middle|\; -\eta_- < \frac{\partial V}{\partial q} < \eta_+\right\}.$$

  Note that in $\mathcal{N}$ the following optimality condition holds:

$$\frac{\partial V}{\partial r} + \beta \frac{V}{q} = \frac{U}{q} - C, \tag{4}$$

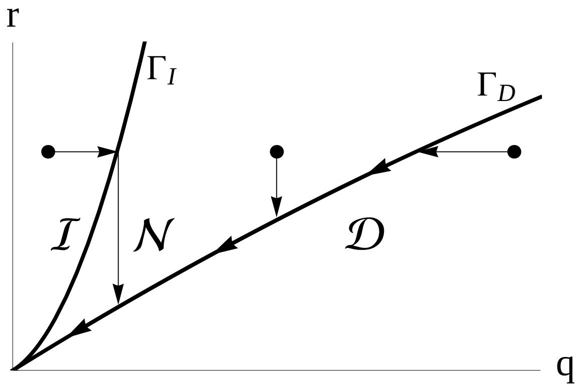
  which states that the marginal value of a resource plus the inflated average value must equal the average utility minus the average extraction costs. Alternatively, *scarcity rent* plus "*average capital rent*" must equal "*average net price*." In fact, the optimal adjustment rule still states an average principle. Here, due to proportional costs, the average rule coincides with marginal rule.

- **Optimal boundaries of upward and downward adjustments**:

$$\Gamma_I := \overline{\mathcal{N}} \cap \overline{\mathcal{I}}, \quad \Gamma_D := \overline{\mathcal{N}} \cap \overline{\mathcal{D}}.$$

Optimal Policy with Adjustment Costs

The optimal adjustment policy can be illustrated by the following figure.



Two free boundaries are not symmetric.

Social Optimal Consumption

Theorem 1

Suppose that $\eta_+ > 0$, *and* $\eta_+ + \eta_- \geqslant 0$, *that the demand function satisfies the following conditions*

$$\int_0^q P(x)dx < \infty \quad \textit{for} \quad q < \infty, \quad \textit{and} \quad P(0) > C + \beta\eta_+.$$

Then there exits a unique socially optimal extraction strategy $\{I_t^*, D_t^*\}_{t\geqslant 0}$ *given by*

$$I_t^* = [Q_I(R_0) - Q_{0^-}]^+, \quad D_t^* = [Q_{0^-} - Q_D(R_0)]^+ + [Q_D(R_0) - Q_D(R_t)]^+,$$

where $Q_I : [0,\infty) \mapsto [0,\infty)$ *and* $Q_D : [0,\infty) \mapsto [0,\infty)$ *are two increasing functions satisfying*

$$Q_I(0) = Q_D(0) = 0, \quad \lim_{r\to\infty} Q_I(r) = q_I, \quad \lim_{r\to\infty} Q_D(r) = q_D,$$
$$\frac{\partial Q_D}{\partial \eta_-} > 0, \quad \frac{\partial Q_I}{\partial \eta_-} < 0, \quad \frac{\partial Q_D}{\partial \eta_+} = 0, \quad \frac{\partial Q_I}{\partial \eta_+} < 0,$$

with $q_I := P^{-1}(C + \beta\eta_+), \quad q_D := \begin{cases} P^{-1}(C - \beta\eta_-) & \textit{if} \quad \eta_- < C/\beta, \\ \infty & \textit{if} \quad \eta_- \geqslant C/\beta. \end{cases}$

Competitive Equilibrium

Having described the socially optimal depletion path, it is natural to ask whether the optimal path can be supported by a system of competitive markets.

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- Without adjustment costs, many authors have shown the equivalence, e.g., *Weinstein and Zeckhauser 1975, QJE*.
- However, *Hartwick et al 1986* show the non-existence of a competitive equilibrium of set-up costs model due to the non-convexity of the economy.
- In contrast, we show the existence of a competitive equilirbium by relating set-up costs with initial adjustments only (i.e. no set up costs afterwards), which still retains the convexity due to the linear structure of adjustment costs.

Competitive Equilibrium

Theorem 2

Suppose that initially there is no production and the total quantity of reserves is $r > 0$, that the conditions in Theorem 1 hold, and that there are many identical price-taking producers.

(i) *There exists a competitive equilibrium which can be characterized by the production rate $\{Q_t^*\}_{t \geqslant 0}$ under the optimal adjustment policy $\{I_t^*, D_t^*\}_{t \geqslant 0}$ given in Theorem 1, and the price $\{p_t^*\}_{t \geqslant 0}$ given as:*

$$p_t^* = \begin{cases} P(Q_I(r)) & \text{for} \quad 0 \leqslant t \leqslant t^*, \\ [P(Q_I(r)) - C + \beta\eta_-]e^{\beta(t-t^*)} + (C - \beta\eta_-) & \text{for} \quad t > t^*, \end{cases}$$

where $t^* := \frac{r - Q_D^{-1}(Q_I(r))}{Q_I(r)}$ is the duration of constant price.

(ii) *The shadow price grows exponetially at rate β. More precisely, $d\lambda_t/dt = \beta\lambda_t$, where $\lambda_t = \partial V/\partial r(R_t, Q_t)$ is the shadow price.*

Competitive Equilibrium Price Profile

Note that for $t > t^*$, the price satisfies that $dp_t/dt = \beta(p_t - C + \beta\eta_-)$. Thus, during the continuously adjusting part, the growth rate of the equilibrium price depends on the sign of η_-.

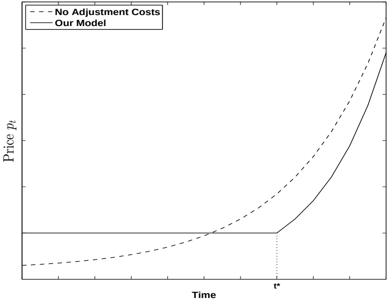

Figure: The competitive equilibrium price with costly downward adjustments in the basic model. The dashed line represents the classic Hotelling's price without adjustment costs.

Extension: Uncertainty, Exploration and Adjustment Costs

- **Demand Uncertainty**: Let us first modify the demand function: $p = X\,P(Q)$. Where X is a non-negative demand factor satisfying

$$dX_t = \mu_x(X_t)dt + \sigma_x(X_t)dB_t^X, \quad X_0 = x,$$

where μ_x and σ_x are functions of X, and B^x is a standard Brownian motion.

- **Exploration Activity**: Following *Pindyck 1978, JPE*, suppose Y_t is the cumulative discoveries at time t, which satisfies

$$dY_t = f(Y_t, W_t)dt, \quad Y_0 = y,$$

where $W_t \geqslant 0$ is the exploratory effort at time t, and f is the discovery rate satisfying $\partial f/\partial Y < 0 < \partial f/\partial W$.
Then the reserve evolves

$$dR_t = dY_t - Q_t dt + \sigma_R(R_t)dB_t^R, \quad R_0 = r,$$

where σ_R is a function, and B^R is a standard Brownian motion correlated with B^X, i.e., $dB_t^X dB_t^R = \rho dt$ for $\rho \in [-1, 1]$.

- Finally, we assume producers are risk neutral and the extraction cost is a decreasing function of reserves $C'(r) < 0 < C(r)$. Also, exploration cost $\tilde{C}(w)$ is increasing and convex.

Extension: Uncertainty and Exploration

Then the social surplus becomes

$$\tilde{S}(\{I_t, D_t, W_t\}) = \int_0^\infty e^{-\beta t}\Big([\tilde{U}(X_t, Q_t) - C(R_t)Q_t - \tilde{C}(W_t)]dt - \eta_+ dI_t - \eta_- dD_t\Big).$$

Consequently,

$$\tilde{V}(r, q, x, y) = \sup_{\{I_t, D_t, W_t\}} \mathbb{E}\tilde{S}(\{I_t, D_t, W_t\}). \tag{5}$$

And the corresponding HJB equation is

$$\max\Big\{\tilde{\mathcal{L}}\tilde{V} + \tilde{U}(x,q) - C(r)q + \max_{w\geqslant 0}\Big(f(w,q)\Big[\frac{\partial \tilde{V}}{\partial r} + \frac{\partial \tilde{V}}{\partial y}\Big] - \tilde{C}(w)\Big), \\ \frac{\partial \tilde{V}}{\partial q} - \eta_+, -\frac{\partial \tilde{V}}{\partial q} - \eta_-\Big\} = 0, \quad \text{for } r \geqslant 0,\ q \geqslant 0,\ x \geqslant 0, y \geqslant 0, \tag{6}$$

where

$$\tilde{\mathcal{L}}\tilde{V} := \frac{1}{2}\sigma_R^2(r)\frac{\partial^2 \tilde{V}}{\partial r^2} + \rho\sigma_R(r)\sigma_X(x)\frac{\partial^2 \tilde{V}}{\partial r \partial x} + \frac{1}{2}\sigma_X^2(x)\frac{\partial^2 \tilde{V}}{\partial x^2} + \mu(x)\frac{\partial \tilde{V}}{\partial x} - q\frac{\partial \tilde{V}}{\partial r} - \beta\tilde{V}.$$

Extension: Uncertainty and Exploration

Theorem 3

Suppose that the conditions in Theorem 1 hold, and that there is a unique classic solution to the HJB equation (6) *with two smooth optimal boundaries* Γ_I *and* Γ_D. *Then,*

(i) *The social optimization problem* (5) *has an optimal production strategy* $\{I_t^*, D_t^*\}_{t\geqslant 0}$ *given respectively by*

$$I_t^* = \int_0^t \mathbf{1}_{\{(R_u^*, Q_u^*, X_u^*, Y_u^*)\in\Gamma_I\}} dI_u^*, \quad D_t^* = \int_0^t \mathbf{1}_{\{(R_u^*, Q_u^*, X_u^*, Y_u^*)\in\Gamma_D\}} dD_u^*,$$

and an optimal exploration effort $\{W_t^*\}_{t\geqslant 0}$ *satisfying*

$$\frac{\partial \tilde{V}(R_u^*, Q_u^*, X_u^*, Y_u^*)}{\partial r} + \frac{\partial \tilde{V}(R_u^*, Q_u^*, X_u^*, Y_u^*)}{\partial y} = \frac{\tilde{C}'(W_t^*)}{\partial f(Y_t^*, W_t^*)/\partial w},$$

with $(R_u^*, Q_u^*, X_u^*, Y_u^*)$ *being the processes under the optimal strategy* $\{I_t^*, D_t^*, W_t^*\}_{t\geqslant 0}$.

(ii) *In addition, this socially optimal consumption path can be reproduced by a competitive market.*

Optimal Boundaries

Suppose that demand shock is temporary which follows a mean-reverting process:

$$\frac{dX_t}{X_t} = \kappa(\mu_T - \log(X_t))dt + \sigma_T dB_t^X,$$

where $\kappa > 0$, $\mu_T > 0$ and $\sigma_T > 0$ are all constants.

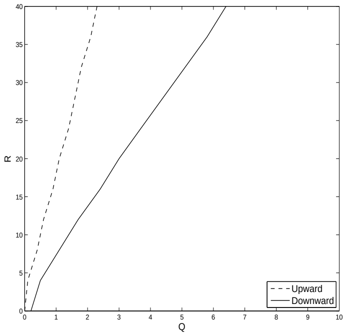

Figure: R-Q Plane

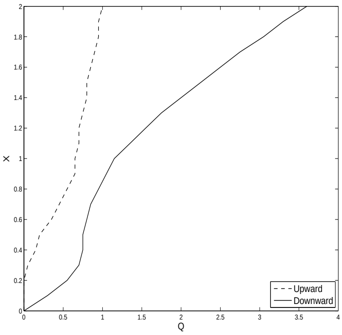

Figure: X-Q Plane

By assuming temporary demand shocks, our model can easily generate many interesting phenomena such as backwardation, cantango, "Samuelson" effect, and high volatility conditional on backwardation (demand uncertainty + other uncertainties (e.g., reserve)).

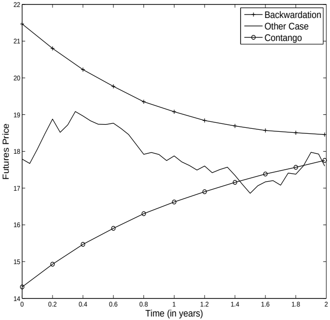

Figure: Futures curves could be either backwarded or in contango.

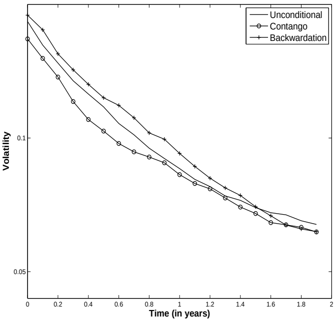

Figure: Samuelson effect and high volatility conditional on backwardation

Long Time Low Price

Finally, either exploration alone or demand uncertainty combined with adjustment costs can generate a U-shaped price profile, while the combination of them will significantly prolong the price staying at the bottom.

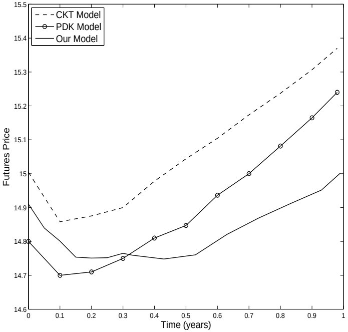

Figure: Parameters are calibrated to CKT's parameters (Short Time)

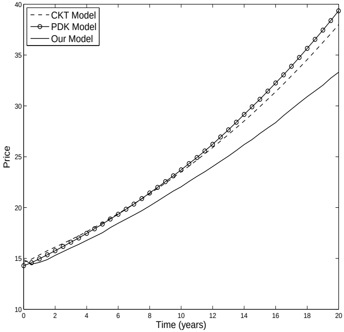

Figure: Parameters are calibrated to CKT's parameters (Long Time)

Long Time Low Price

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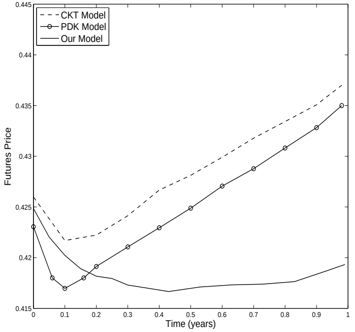

Figure: Robustness: Other Parameters (Short Time)

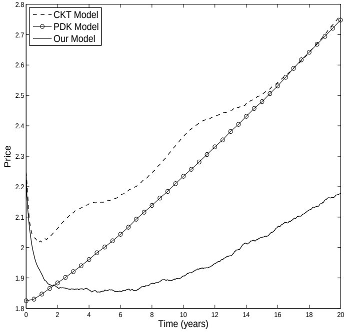

Figure: Robustness: Other Parameters (Long Time)

Conclusion

- We have proposed a general equilibrium model of exhaustible resources with production adjustment costs, which is analytically tractable.
- The adjustment costs combined with demand uncertainty can generate many interesting phenomena such as backwardation, contango, and Samuelson effect.
- If, in addition, the exploration is added into consideration, our model shows that the volatility of futures prices is higher conditional on backwardation.
- The combination of both exploration and adjustment costs can significantly prolong the time periods of low prices.

Thank you very much!