Life-Cycle Consumption, Investment, and Voluntary Retirement with Cointegration between the Stock and Labor Markets

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Motivation

- Empirical Literature
  - **Cointegration** between the stock and labor markets exists
    - Baxter and Jerman (1997, AER), Campbell (1996, JPE), Menzly *et al.* (2004, JPE), Santos and Veronesi (2006, RFS)
  - **Proportion of financial wealth (hereafter, portfolio share) invested in stocks increases with financial wealth** (Survey of Consumer Finances; Calvet and Sodini, 2014, JF; Kim *et al.*, 2016, JFE)
  - **Early retirement** was quite possible between 1995 and 2000 in which the U.S. economy experienced a stock market boom with a rapid increase in the stock market returns
    - Issues in Labor Statistics (Bureau of Labor Statistics, 2000), Gustman and Steinmeier (2002), Gustman *et al.* (2010, JEP)

Motivation (cont'd)

- Theoretical Literature
  - Life-cycle portfolio choice implications **in the presence of cointegration**, but **without voluntary (or endogenous) retirement**
    - Benzoni *et al.* (2007, JF)
  - **Increasing portfolio share with wealth (or age)** has been observed by
    - Farhi and Panageas (2007, JFE): voluntary (or endogenous) labor-supply flexibility along the extensive margin
    - Polkovnichenko (2007, RFS): additive and endogenous habit formation preferences
    - Wachter and Yogo (2010, RFS): a nonhomothetic utility over basic and luxury goods for households
    - Dybvig and Liu (2010, JET): non-spanned income risks with very high correlation with stock market
  - There has been **no consensus** as to the economic rationality for early retirement during stock market booms
    - Bensoussan *et al.* (2016, OR): counter-cyclical patterns of the number of unemployed job leavers

Motivation (cont'd)

- Dybvig and Liu (2010) claim in the conclusion of their paper as follows.

  It would be nice to add more state variables to the model. For example, it has long been known that wages are sticky and it is reasonable that they respond to shocks in the stock market, but with a delay. ... Unfortunately, models with additional state variables seem almost impossible to be solved analytically given current tools and numerical solution is also very difficult.

- Extension of Dybvig and Liu (2010) with cointegration between the stock and labor markets.

Two Main Results

- There exists a target wealth-to-income ratio under which an investor does not participate in the stock market at all (**non-participation puzzle**), whereas above which **the investor increases the proportion of financial wealth invested in the stock market as she accumulates wealth** (consistent with empirical observations).
- **Early retirement is quite reasonable and even numerically plausible during stock market booms** (consistent with empirical facts), especially when wages are expected to decline in the long term.

Model Setup

- Consider an infinitely-lived investor with:
  - uninsurable labor income risks with (1) diffusive and continuous shock, and (2) discrete and jump shock
  - cointegration between the stock and labor markets
  - borrowing and short sale constraints
  - a constant investment opportunity set
  - standard constant relative risk aversion (CRRA) utility preference
  - retirement flexibility

Labor Income Process (without cointegration)

- Consider the following widely used a geometric Brownian motion process with a exogenously-driven Poisson jump:

$$dI_t = \mu_I I_{t-} dt + \sigma_I I_{t-} dB_t^3 - (1-\kappa) I_{t-} dN_t, \quad I_0 = I > 0, \quad \kappa \in [0, 1].$$

  - N_t is a pure Poisson jump process with intensity δ_D.
    - When a discrete and jump shock happens at time τ_D, labor income decreases from $I_{\tau_D -}$ to $\kappa I_{\tau_D -}$.
  - **Risks associate with labor income are uninsurable or unhedgeable because of non-tradability and market incompleteness**.

Financial Market

- An individual can invest her savings in two assets: a riskless bond and a risky stock
  - the bond price grows at a constant rate $r > 0$.
  - the stock price, S_t, follows a geometric Brownian motion:

$$dS_t = \mu S_t dt + \sigma S_t dB_t^1.$$

  - B_t^1 is a standard one-dimensional Brownian motion with the instantaneous correlation $\rho \in [-1, 1]$ with the labor income process.
  - **A partial hedging effect against income risks by investing in the stock market.**

Cointegration between the Stock and Labor Markets

- One more state variable to capture cointegration between the stock and labor markets. Specifically,

$$I_t = S_t e^{Z_t}, \text{ or equivalently}, Z_t = \ln I_t - \ln S_t,$$

$$dI_t/I_{t-} = \{\mu_I - \alpha(Z_t - \overline{z})\}dt + (\sigma - \sigma_z)dB_t^1 + \sigma_I dB_t^2 - (1-\kappa)dN_t, \quad I_0 = I > 0,$$

where

$$\mu_I = \mu + \frac{1}{2}\sigma_z^2 + \frac{1}{2}\sigma_I^2 - \sigma\sigma_z.$$

  - Empirically plausible parameter assumption ($\sigma = \sigma_z$): contemporaneous (or instantaneous) correlation between income risks and stock returns should be zero (Davis and Willen, 2000; Cocco *et al.*, 2005, RFS).
  - When $Z_t - \overline{z} < 0$, the labor income will be increased in the long term, whereas when $Z_t - \overline{z} > 0$, the labor income decreased.

Short Sale and Borrowing Constraints

- **While Benzoni *et al.* (2007) do now allow for short sale and borrowing constraints, this paper considers both constraints.**
- We require that both stock investment y_t and bond investment x_t should be nonnegative, i.e.,

$$y_t \geq 0, \quad x_t \geq 0,$$

which evolves by the following dynamics:

$$dW_t = (rW_t - c_t + l_t)dt + y_t\sigma(dB_t + \theta dt), \quad W_0 = w \geq 0.$$

- Let $\mathcal{A}(w, l, z)$ denote the set of admissible policies (c_t, y_t) such that short sale and borrowing constraints are satisfied.

A Consumption, Investment, and Voluntary Retirement Problem

- An individual's life-cycle problem (based on expected utility theory) is to maximize her CRRA expected utility preference by controlling per-period consumption c, risky investment y, and voluntary retirement time τ:

$$V(w,I,z) = \sup_{(c,y,\tau)\in\mathcal{A}(w,I,z)} \mathbb{E}\Big[\int_0^{\tau} e^{-(\beta+\delta_D)t}\Big\{\frac{c_t^{1-\gamma}}{1-\gamma} + \delta_D V(W_t,\kappa I_t,Z_t)\Big\}dt + e^{-(\beta+\delta_D)\tau}\int_{\tau}^{\infty}\frac{(Bc_t)^{1-\gamma}}{1-\gamma}dt,$$

where $B > 1$ stands for the leisure preference after voluntary retirement.

Variational Inequality

- It turns out that the value function should satisfy the variational inequality given by (Bensoussan and Lions, 1982; Øksendal, 2007): for any $w \geq 0$, $l \geq 0$, $z \in \mathbb{R}$,

$$\begin{cases} \max_{(c,y)\in\mathcal{A}(w,l)} \mathcal{L}V(w,l,z) \leq 0, \\ V(w,l,z) \geq R(w), \\ \max_{(c,y)\in\mathcal{A}(w,l)} \mathcal{L}V(w,l,z) \times \Big\{R(w) - V(w,l,z)\Big\} = 0, \end{cases}$$

where the differential operator $\mathcal{L}$ and function $R(w)$ are given by

$$\begin{aligned} \mathcal{L}V =& f(c,V) - cV_w + \frac{1}{2}\sigma^2 y^2 V_{ww} + \frac{1}{2}[\sigma_l^2 + (\sigma - \sigma_z)^2] l^2 V_{ll} + \frac{1}{2}(\alpha_z^2 + \sigma_l^2) V_{zz} \\ & + \sigma(\sigma - \sigma_z) y l V_{wl} - \sigma\sigma_z y V_{wz} + [\sigma_l^2 - (\sigma - \sigma_z)\sigma_z] l V_{zl} \\ & + [y(\mu - r) + rw + l] V_w + [\mu_l - \alpha(z - \overline{z})] l V_l - \alpha(z - \overline{z}) V_z \\ & - \delta_D \mathbb{E}\Big[V(w,l,z) - V(w,\kappa l,z)\Big], \end{aligned}$$

$$R(w) = \frac{B^{1-\gamma}\overline{K}^{-\gamma}}{1-\gamma} w^{1-\gamma}, \quad \overline{K} = \frac{\beta}{\gamma} - \frac{1-\gamma}{\gamma}\Big(r + \frac{\theta^2}{2\gamma}\Big).$$

Verification and Homogeneous Tranformation

- Verification: optimal consumption, investment, and retirement strategies are

$$\begin{aligned} c_t^* &= V_w(W_t, I_t, Z_t)^{-1/\gamma}, \\ y_t^* &= \max\left\{\min\left\{\frac{\sigma_z V_{wz}(W_t, I_t, Z_t) - (\sigma - \sigma_z) I_t V_{wI}(W_t, I_t, Z_t)}{\sigma V_{ww}(W_t, I_t, Z_t)}, 1\right\}, 0\right\}, \\ \tau^* &= \inf\{t \geq 0 : V(W_t, I_t, Z_t) \geq R(W_t)\}. \end{aligned}$$

- Homogeneity property $V(\lambda w, \lambda I, z) = \lambda^{1-\gamma} V(w, I, z)$:

$$V(w, I, z) = \frac{\bar{K}^{-\gamma}}{1-\gamma}\left(w + \frac{I}{r}\right)^{1-\gamma} e^{(1-\gamma)u(\xi, z)}, \quad \xi = \frac{I/r}{w + I/r} \in [0, 1].$$

Verification and Homogeneous Tranformation

- New value function $u(\zeta, z)$ satisfies HJB:

$$\max_{\bar{y},\bar{c}} \{\mathcal{L}_1 u(\xi, z), \quad \mathcal{R}u(\xi)\} = 0, \tag{1}$$

on $\{(\xi, z) : \xi \in [0, 1], z \in \mathbb{R}\}$ where

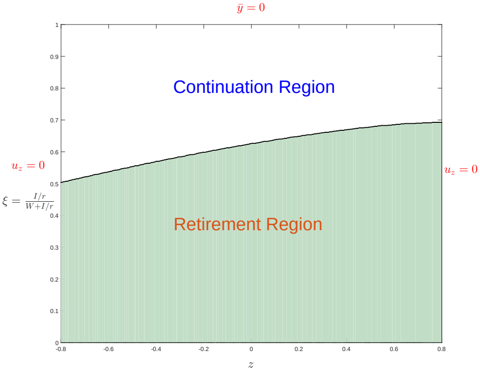
$$\begin{aligned}
\mathcal{L}_1 u = & \left[\frac{1}{2}\sigma^2\bar{y}^2\xi^2 + \frac{1}{2}(\sigma_I^2 + (\sigma - \sigma_Z)^2)\xi^2(1-\xi)^2 - \sigma(\sigma - \sigma_Z)\bar{y}\xi^2(1-\xi)\right][u_{\xi\xi} + (1-\gamma)u_\xi^2] \\
& + \left[\sigma\sigma_Z\bar{y}\xi + (\sigma_I^2 - \sigma_Z(\sigma - \sigma_Z))\xi(1-\xi)\right][u_{\xi z} + (1-\gamma)u_\xi u_z] + \frac{1}{2}(\sigma_I^2 + \sigma_Z^2)[u_{zz} + (1-\gamma)u_z^2] \\
& + \left[\gamma\sigma^2\bar{y}^2 + \gamma\sigma(\sigma - \sigma_Z)(2\xi - 1)\bar{y} - (\mu - r)\bar{y} - \gamma(\sigma_I^2 + (\sigma - \sigma_Z)^2)\xi(1-\xi)\right]\xi u_\xi \\
& + [(\mu_I - \alpha(z - \bar{z}))(1-\xi) - r]\xi u_\xi + \left[-(1-\gamma)\sigma\sigma_Z\bar{y} + (1-\gamma)(\sigma_I^2 - \sigma_Z(\sigma - \sigma_Z))\xi - \alpha(z - \bar{z})\right]u_z \\
& + (\mu - r - \gamma\sigma(\sigma - \sigma_Z)\xi)\bar{y} - \frac{1}{2}\gamma\sigma^2\bar{y}^2 - \frac{1}{2}(\sigma_I^2 + (\sigma - \sigma_Z)^2)\gamma\xi^2 + (\mu_I - \alpha(z - \bar{z}))\xi + r - \frac{\beta + \delta_D}{1-\gamma} \\
& + \delta_D\frac{(1 + (\kappa - 1)\xi)^{1-\gamma}}{1-\gamma}E\left[e^{(1-\gamma)\left(u\left(\frac{\kappa\xi}{1+(\kappa-1)\xi}\right) - u\right)}\right] + \frac{\bar{K}^\gamma}{1-\gamma}e^{-(1-\gamma)u}\bar{c}^{1-\gamma} - \bar{c}(1 - \xi u_\xi),
\end{aligned}$$

$$\mathcal{R}u = \ln(1 + (\kappa - 1)x) + \ln B - u,$$

Boundary Condition

- At boundary $\xi = 1$, i.e., when $w = 0$, it is known that the investor should invest no stocks and only consume part of the income:

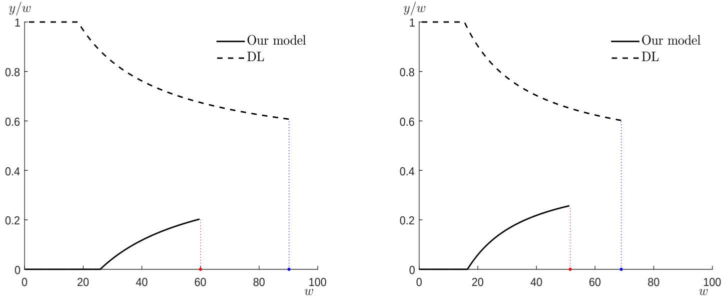
$$\bar{y}^* = 0,$$
$$\bar{c}^* = \min\left\{\bar{K}e^{(1-1/\gamma)u(1,z)}(1 - u_\xi(1,z))^{-1/\gamma}, r\right\}.$$



Baseline Parameters

- Asset returns (Dybvig and Liu, 2010)
  - $\mu = 5\%$ (expected rate of stock return), $r = 1\%$ (risk-free interest rate), $\sigma = 18\%$ (stock volatility)
- Annual subjective discount rate (Cocco *et al.*, 2005, RFS; Gomes and Michaelides, 2005, JF; Wachter and Yogo, 2010, RFS; Wang *et al.*, 2016, JET): $\beta = 4\%$
- Relative risk aversion: $\gamma = 3$
- Leisure preference after voluntary retirement: $B = 2$
- Parameters in labor income process (Deaton, 1991, Econometrica; Carroll, 1997, QJE; Benzoni *et al.*, 2007, JF; Wang *et al.*, 2016, JET):
  - $\mu_I = 0.5\%$ (expected rate of income growth), $\sigma_I = 10\%$ (volatility on income growth), $\alpha = 15\%$ (degree of mean reversion), $\overline{z} = 0$ (long-term mean), $I = 1$ (annual rate of labor income), $\kappa = 80\%$ (recovery parameter), $\delta_D = 0.5\%$ (disastrous labor income shock intensity)
  - We take the empirically plausible parameter assumption that the contemporaneous correlation between labor income shocks and market returns should be zero, i.e., we set $\sigma = \sigma_Z$.

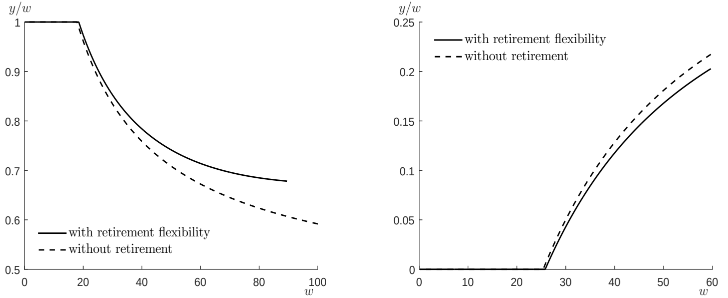
Portfolio Share



Left Panel (without jumps in income) and Right Panel (with jumps in income)

- A target wealth-to-income ratio: under which non-participation in the stock market and above which increasing portfolio share with wealth.
- Human capital's implicit equity holdings (with little wealth) versus implicit bond holdings (with large wealth).

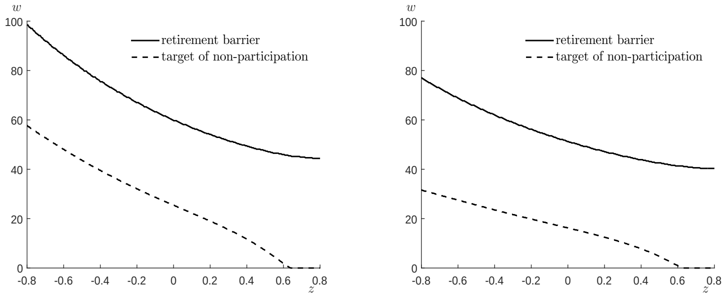
Effect of Retirement Flexibility on Portfolio Share



Left Panel (without cointegration) and Right Panel (with cointegration)

- Without cointegration, retirement flexibility (a real option) is found to increase equity holdings.
- With cointegration, the result is reversed because retirement flexibility makes labor income's beta more positive rather than negative.

Economic Plausibility of Early Retirement



Left Panel (without jumps in income) and Right Panel (with jumps in income)

- It is optimal for individuals to enter voluntary retirement as soon as their wealth approaches a certain wealth threshold: American style call option.
- Early retirement is numerically plausible in terms of the wealth threshold especially when labor income will be decreased (i.e., $z > 0$) than when it will be increased (i.e., $z < 0$).

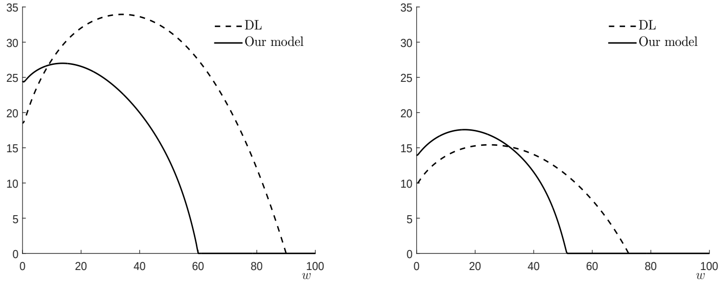
Economic Plausibility of Early Retirement (cont'd)

- Implicit value of labor income:

$$\frac{\partial V(w,l,z)}{\partial l} \Big/ \frac{\partial V(w,l,z)}{\partial w}.$$

- That is, human capital's implicit value is an individual's subjective marginal value of her future labor income and can become a proxy for the individual's early retirement demand.
- When an investor accumulates wealth, which one is rational (or optimal) to work more or less in terms of marginal utility values?

Economic Plausibility of Early Retirement (cont'd)



Left Panel (without jumps in income) and Right Panel (with jumps in income)

- Our model generates the empirically plausible hump shape of implicit value of human capital.
- Cointegration leads to a earlier peak point in the implicit value compared to Farhi and Panageas (2007) and Dybvig and Liu (2010) without cointegration.

One Page Conclusion

- (Academic) We develop a new life-cycle model through which three empirical observations can be explained:
  - non-participation in the stock market
  - increasing portfolio share with wealth
  - retiring early during stock market booms
- (Industry) How can this model be used to solve real-world economic problems such as global retirement funding problem in the short and long runs?
  - Global trend: worldwide retirement systems allows more freedom (or flexibility, or real options) to individuals when investing and retiring.
  - Long-run dependence (cointegration) between income returns and stock market performance should be considered when funding retirement plans.