Demand for Life Insurance of a Family with Working Couple

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Motivation

- Our model: we develop a continuous-time model to investigate an optimal consumption, investment, and life insurance decision of a family with a **working couple** (two wage earners).
- Life insurance is an instrument to hedge the mortality risk (the death of wage earner) to lose the future labor income of wage earner.
- Thus, it is natural that there is a positive relation between your expected future income and the demand for life insurance on you. (several empirical studies support this)
- Question: If your spouse's (not your) expected future income increases, then how the demand for life insurance on you changes?

- In the last paragraph of conclusion of Fitzgerald (1987, JRI):
  "The results also suggest that households do not view the future earnings of the wife as decreasing the need for life insurance on the husband. This runs counter to the prediction of the model presented, and suggests the need for further work on modeling risk sharing within households, particularly on how wives' future earnings and husbands' future earnings are interrelated."
  - Fitzgerald (1987, JRI)'s model (one period, utility maximizing model) predicts negative relation between wife's future earning and the life insurance on the husband.
  - However, the data in Fitzgerald (1987, JRI) shows positive relation, which is the opposite of the prediction of their model.

- Our model suggests that the relation between your spouse's future earning and the demand for life insurance on you can be positive or negative depending on the bequest motive.
- We believe that our model also has potential to explain other empirical findings on the determinants of the demand for life insurance. (this is an ongoing project)

Financial Market Model

- Continuous time financial market model
- two assets: a risk-free asset (with constant interest rate r) and a risky asset $S(t)$,
$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dW(t),$$
- $W(t)$: standard Brownian motion, constant μ, σ
- Market price of risk $\theta = \frac{\mu - r}{\sigma}$.

Working Couple

- There are two wage earners in the family:
  wage earner 1 and wage earner 2
  (for example, wage earner 1: husband, wage earner 2: wife)

- deterministic income streams:
  $\epsilon_1(t)$ (wage earner 1's income) and $\epsilon_2(t)$ (wage earner 2's income)

- They consume separately, but share the financial wealth:
  - wage earner 1's consumption rate: $c_1(t)$
  - wage earner 2's consumption rate: $c_2(t)$
  - family's risky investment (amount): $\pi(t)$
  - family's wealth process: $X(t)$

Uncertain Lifetime

- Uncertain lifetime of wage earners: mortality risk
- wage earner 1's death time: τ_1, wage earner 2's death time: τ_2
- wage earner 1's mortality intensity: $\lambda_1(t)$, wage earner 2's mortality intensity: $\lambda_2(t)$
  - $\mathbb{P}(\tau_1 > t) = e^{-\int_0^t \lambda_1(u)du} ds$
  - $\mathbb{P}(\tau_2 > t) = e^{-\int_0^t \lambda_2(u)du} ds$
- Assume τ_1 and τ_2 are independent, and they are independent of financial risk
- Life insurance contract is necessary to hedge the mortality risk
- For simplicity of presentation, we focus on the case with constant λ_1 and λ_2 from now on.

Life Insurance

- If the wage earner 1 pays life insurance premium $p_1(t)$, then at the death time of wage earner 1, the insurance benefit is $p_1(\tau_1-)/\lambda_1$,

$$X(\tau_1) = X(\tau_1-) + \frac{p_1(\tau_1-)}{\lambda_1}$$

- If the wage earner 2 pays life insurance premium $p_2(t)$, then at the death time of wage earner 2, the insurance benefit is $p_2(\tau_2-)/\lambda_2$,

$$X(\tau_2) = X(\tau_2-) + \frac{p_2(\tau_2-)}{\lambda_2}$$

- We may consider positive premium loading, i.e.,

$$X(\tau_i) = X(\tau_i-) + \frac{p_i(\tau_i-)}{\eta_i},$$

where $\eta_i > \lambda_i$. But we focus on zero premium loading case, $\eta_i = \lambda_i$.

Wealth Process

- For $0 \le t < \tau_1 \vee \tau_2 \triangleq \max(\tau_1, \tau_2)$ (while at least one of wage earner 1 and wage earner 2 is alive)

$$\begin{aligned} dX(t) = \Big[& rX(t) + (\mu - r)\pi(t) \\ & + \mathbf{1}_{\{t<\tau_1\}}(-c_1(t) - p_1(t) + \epsilon_1(t)) \\ & + \mathbf{1}_{\{t<\tau_2\}}(-c_2(t) - p_2(t) + \epsilon_2(t))\Big] dt + \sigma\pi(t)dW(t) \end{aligned} \quad (1)$$

- At the death time of wage earner 1, τ_1:

$$X(\tau_1) = X(\tau_1-) + \frac{p_1(\tau_1-)}{\lambda_1} \quad (2)$$

- At the death time of wage earner 2, τ_2:

$$X(\tau_2) = X(\tau_2-) + \frac{p_2(\tau_2-)}{\lambda_2} \quad (3)$$

Family's Optimization Problem

- (Main Problem) Family's optimization:

$$V(x) = \max_{c_1,c_2,p_1,p_2,\pi} \mathbb{E}\Big[\alpha_1 \int_0^{\tau_1} e^{-\beta t} U_1(c_1(t))dt + \alpha_2 \int_0^{\tau_2} e^{-\beta t} U_2(c_2(t))dt + \alpha_3 e^{-\beta(\tau_1 \vee \tau_2)} U_3(X_{\tau_1 \vee \tau_2}) \Big| X(0) = x\Big]$$

  subject to (1), (2), and (3).

- – $U_1(\cdot)$: utility function of wage earner 1's consumption
  – $U_2(\cdot)$: utility function of wage earner 2's consumption
  – $U_3(\cdot)$: bequest function (utility of bereaved family)

- α_i for $i \in \{1, 2, 3\}$ are weights for the utilities of wage earner 1, wage earner 2, and bereaved family, respectively.

- $\beta > 0$: subjective discount rate

- We consider CRRA(constant relative risk aversion) utility functions:

$$U_i(x) = \frac{1}{1-\gamma_i} x^{1-\gamma_i}, \quad \gamma_i > 0,\ \gamma_i \neq 1, \quad i \in \{1,2,3\}.$$

- Note that

$$\begin{aligned}
&\mathbb{E}\Big[\alpha_1 \int_0^{\tau_1} e^{-\beta t} U_1(c_1(t))dt + \alpha_2 \int_0^{\tau_2} e^{-\beta t} U_2(c_2(t))dt + \alpha_3 e^{-\beta(\tau_1 \vee \tau_2)} U_3(X_{\tau_1 \vee \tau_2})\Big] \\
&= \underbrace{\mathbb{E}\Big[\alpha_1 \int_0^{\tau_1 \wedge \tau_2} e^{-\beta t} U_1(c_1(t))dt + \alpha_2 \int_0^{\tau_1 \wedge \tau_2} e^{-\beta t} U_2(c_2(t))dt}_{\text{both wage earners are alive}} \\
&\quad + \underbrace{\mathbf{1}_{\{\tau_1 \geq \tau_2\}} \mathbb{E}_{\tau_2} \left[\alpha_1 \int_{\tau_2}^{\tau_1} e^{-\beta t} U_1(c_1(t))dt + \alpha_3 e^{-\beta(\tau_1)} U_3(X_{\tau_1})\right]}_{\text{only wage earner 1 is alive}} \\
&\quad + \underbrace{\mathbf{1}_{\{\tau_1 < \tau_2\}} \mathbb{E}_{\tau_1} \left[\alpha_2 \int_{\tau_1}^{\tau_2} e^{-\beta t} U_2(c_2(t))dt + \alpha_3 e^{-\beta(\tau_2)} U_3(X_{\tau_2})\right]}_{\text{only wage earner 2 is alive}} \Big]
\end{aligned}$$

Sub Problems (only one wage earner remains)

- Optimization problem when only one wage earner is alive:
  if $\tau_i > \tau_j, i \neq j, i, j \in \{1,2\}$ and for $\tau_j \leq s < \tau_i$,

$$V_i(x,s) = \max_{c_i(t), p_i(t), \pi(t)} \mathbb{E}_s \Big[\alpha_i \int_s^{\tau_i} e^{-\beta(t-s)} U_i(c_i(t)) dt + \alpha_3 e^{-\beta(\tau_i - s)} U_3 \Big(X_{\tau_i -} + \frac{p_i(\tau_i -)}{\lambda_i} \Big) \Big| X_s = x \Big],$$

subject to

$$dX(t) = \left[rX(t) + (\mu - r)\pi(t) - c_i(t) - p_i(t) + \epsilon_i(t) \right] dt + \sigma \pi(t) dW(t), \quad X_s = x.$$

- $V_1(x,s)$: only wage earner 1 is alive,
  $V_2(x,s)$: only wage earner 2 is alive.

- We find $V_1(x,s)$ and $V_2(x,s)$ first, then use them to rewrite the value function $V(x)$.

Solutions to Sub Problems

- We apply the martingale approach to find the solutions to $V_i(x,s)$
- The solution to $V_i(x,s)$ is as follows:

$$V_i(x,s) = \frac{1}{1-\gamma_i}\frac{\alpha_i^{\frac{1}{\gamma_i}}}{\lambda_i + K_i} {v_i^*}^{1-\frac{1}{\gamma_i}} + \frac{1}{1-\gamma_3}\frac{\lambda_i \alpha_3^{\frac{1}{\gamma_3}}}{\lambda_i + K_3} {v_i^*}^{1-\frac{1}{\gamma_3}},$$

where v_i^* is the unique root of the following algebraic equation:

$$\frac{\alpha_i^{\frac{1}{\gamma_i}}}{\lambda_i + K_i} {v_i^*}^{-\frac{1}{\gamma_i}} + \frac{\lambda_i \alpha_3^{\frac{1}{\gamma_3}}}{\lambda_i + K_3} {v_i^*}^{-\frac{1}{\gamma_3}} = x + b_i(s).$$

- $b_i(s)$, the present value of agent i's cumulative future labor income:

$$b_i(s) \triangleq \mathbb{E}_s\left[\int_s^\infty e^{-\lambda_i(t-s)}\frac{H(t)}{H(s)}\epsilon_i(t)dt\right],$$

where $H(t) = \exp\left\{-\left(r + \frac{1}{2}\theta^2\right)t - \theta W(t)\right\}$.

- K_i for $i \in \{1,2,3\}$ are defined as $K_i = r + \frac{\beta - r}{\gamma_i} + \frac{\gamma_i - 1}{2\gamma_i^2}\theta^2 > 0$.

Main Problem with $V_1(x,s)$ and $V_2(x,s)$

- Family's optimization problem can be rewritten as follows:

$$V(x) = \max_{c_1,c_2,p_1,p_2,\pi} \mathbb{E}\Big[\underbrace{\alpha_1 \int_0^{\tau_1\wedge\tau_2} e^{-\beta t} U_1(c_1(t))dt + \alpha_2 \int_0^{\tau_1\wedge\tau_2} e^{-\beta t} U_2(c_2(t))dt}_{\text{both wage earners are alive}}$$

$$+ \underbrace{\mathbf{1}_{\{\tau_1 \geq \tau_2\}} e^{-\beta\tau_2} V_1(M_2(\tau_2), \tau_2)}_{\text{only wage earner 1 is alive}}$$

$$+ \underbrace{\mathbf{1}_{\{\tau_1 < \tau_2\}} e^{-\beta\tau_1} V_2(M_1(\tau_1), \tau_1)}_{\text{only wage earner 2 is alive}} \Big| X(0) = x \Big]$$

subject to

$$dX(t) = \Big[rX(t) + (\mu - r)\pi(t) + (-c_1(t) - p_1(t) + \epsilon_1(t)) + (-c_2(t) - p_2(t) + \epsilon_2(t)) \Big] dt$$
$$+\sigma\pi(t)dW(t), \qquad \text{for } 0 \leq t < \tau_1 \wedge \tau_2,$$

where

$$M_1(t) = X(t-) + \frac{p_1(t-)}{\lambda_1}, \quad M_2(t) = X(t-) + \frac{p_2(t-)}{\lambda_2}.$$

- Recall that τ_1 and τ_2 are independent, and
  - $\mathbb{P}(\tau_1 > t) = e^{-\int_0^t \lambda_1(u)du} ds$
  - $\mathbb{P}(\tau_2 > t) = e^{-\int_0^t \lambda_2(u)du} ds$.
- Moreover, τ_1 and τ_2 are independent of financial risk. Thus, we have the following:

  For $i \neq j$, where $i, j \in \{1, 2\}$, the following equations hold:

  (a) $\mathbb{E}\left[\mathbf{1}_{\{\tau_i > \tau_j\}} e^{-\beta\tau_j} V_i(M_{\tau_j}, \tau_j)\right] = \mathbb{E}\left[\int_0^\infty \lambda_j e^{-(\beta+\lambda_1+\lambda_2)t} V_i(M_j(t), t) dt\right]$.

  (b) $\mathbb{E}\left[\mathbf{1}_{\{\tau_i < \tau_j\}} e^{-\beta\tau_i} V_j(M_{\tau_i}, \tau_i)\right] = \mathbb{E}\left[\int_0^\infty \lambda_i e^{-(\beta+\lambda_1+\lambda_2)t} V_j(M_i(t), t) dt\right]$.

  (c) $\mathbb{E}\left[\int_0^{\tau_1 \wedge \tau_2} e^{-\beta t} U_i(c_i(t)) dt\right] = \mathbb{E}\left[\int_0^\infty e^{-(\beta+\lambda_1+\lambda_2)t} U_i(c_i(t)) dt\right]$

- Note that the discount rates in the right-hand sides of (a), (b), (c) are equal.

Main Problem (restated)

- The value function $V(x)$ becomes

$$V(x) = \max_{c_1,c_2,p_1,p_2,\pi} \mathbb{E}\Big[\int_0^\infty e^{-(\beta+\lambda_1+\lambda_2)t} \left(\alpha_1 U_1(c_1(t)) + \alpha_2 U_2(c_2(t))\right) dt + \int_0^\infty \lambda_2 e^{-(\beta+\lambda_1+\lambda_2)t} V_1(M_2(t),t) dt + \int_0^\infty \lambda_1 e^{-(\beta+\lambda_1+\lambda_2)t} V_2(M_1(t),t) dt \Big]$$

subject to

$$\mathbb{E}\left[\int_0^\infty e^{-(\lambda_1+\lambda_2)t} H(t)(c_1(t)+c_2(t)-\epsilon_1(t)-\epsilon_2(t)) dt + \int_0^\infty e^{-(\lambda_1+\lambda_2)t} (\lambda_1 H(t) M_1(t) + \lambda_2 H(t) M_2(t)) dt \right] \le x.$$

Solution to $V(x)$

- The value function $V(x)$ is given as follows:

$$V(x) = \alpha_1^{\frac{1}{\gamma_1}} \frac{\gamma_1}{1-\gamma_1} \frac{1}{\lambda_1 + K_1} {v^*}^{1-\frac{1}{\gamma_1}} + \alpha_2^{\frac{1}{\gamma_2}} \frac{\gamma_2}{1-\gamma_2} \frac{1}{\lambda_2 + K_2} {v^*}^{1-\frac{1}{\gamma_2}}$$
$$+ \alpha_3^{\frac{1}{\gamma_3}} \frac{\gamma_3}{1-\gamma_3} \left(\frac{\lambda_1\lambda_2}{\lambda_1 + K_3} + \frac{\lambda_1\lambda_2}{\lambda_2 + K_3} \right) \frac{1}{\lambda_1 + \lambda_2 + K_3} {v^*}^{1-\frac{1}{\gamma_3}},$$

where v^* satisfies the following algebraic equation:

$$x + b_1(0) + b_2(0) = \alpha_1^{\frac{1}{\gamma_1}} \frac{1}{\lambda_1 + K_1} {v^*}^{-\frac{1}{\gamma_1}} + \alpha_2^{\frac{1}{\gamma_2}} \frac{1}{\lambda_2 + K_2} {v^*}^{-\frac{1}{\gamma_2}}$$
$$+ \alpha_3^{\frac{1}{\gamma_3}} \left(\frac{\lambda_1\lambda_2}{\lambda_1 + K_3} + \frac{\lambda_1\lambda_2}{\lambda_2 + K_3} \right) \frac{1}{\lambda_1 + \lambda_2 + K_3} {v^*}^{-\frac{1}{\gamma_3}}.$$

Optimal Life Insurance Premium

- Optimal life insurance premium of wage earner 1:

$$p_1^*(t) = \lambda_1 \left\{ b_1(t) - \alpha_1^{\frac{1}{\gamma_1}} \frac{1}{\lambda_1 + K_1} y^*(t)^{-\frac{1}{\gamma_1}} + \alpha_3^{\frac{1}{\gamma_3}} \frac{\lambda_2 K_3}{(\lambda_1 + K_3)(\lambda_1 + \lambda_2 + K_3)} y^*(t)^{-\frac{1}{\gamma_3}} \right\},$$

- Optimal life insurance premium of wage earner 2:

$$p_2^*(t) = \lambda_2 \left\{ b_2(t) - \alpha_2^{\frac{1}{\gamma_2}} \frac{1}{\lambda_2 + K_2} y^*(t)^{-\frac{1}{\gamma_2}} + \alpha_3^{\frac{1}{\gamma_3}} \frac{\lambda_1 K_3}{(\lambda_2 + K_3)(\lambda_1 + \lambda_2 + K_3)} y^*(t)^{-\frac{1}{\gamma_3}} \right\}.$$

- $y^*(t) = v^* e^{-\beta(t-s)} \frac{H(t)}{H(s)}$ is the dual variable, where v^* is determined in the previous page.

- The relation between $y^*(t)$ and $X(t)$ is as follows:

$$X(t) = \alpha_1^{\frac{1}{\gamma_1}} \frac{1}{\lambda_1 + K_1} y^*(t)^{-\frac{1}{\gamma_1}} + \alpha_2^{\frac{1}{\gamma_2}} \frac{1}{\lambda_2 + K_2} y^*(t)^{-\frac{1}{\gamma_2}}$$
$$+ \alpha_3^{\frac{1}{\gamma_3}} \left(\frac{\lambda_1 \lambda_2}{\lambda_1 + K_3} + \frac{\lambda_1 \lambda_2}{\lambda_2 + K_3} \right) \frac{1}{\lambda_1 + \lambda_2 + K_3} y^*(t)^{-\frac{1}{\gamma_3}} - b_1(t) - b_2(t),$$

Finding 1. Sign of $\dfrac{\partial p_1^*(t)}{\partial b_1(t)}$ and $\dfrac{\partial p_2^*(t)}{\partial b_2(t)}$

- Regardless of α_3, λ_1, and λ_2, as we expected, we have

$$\frac{\partial p_1^*(t)}{\partial b_1(t)} > 0, \quad \frac{\partial p_2^*(t)}{\partial b_2(t)} > 0. \tag{4}$$

  - In other words, if your (resp. your spouse's) income <u>increases</u>, then it is optimal to <u>purchase more</u> life insurance on you (resp. your spouse) to hedge the loss of human capital at the death time.

- Question: If your spouse's income increases, then is it optimal to increase or decrease the life insurance premium on you? In other words, what is the sign of $\frac{\partial p_1^*(t)}{\partial b_2(t)}$ and $\frac{\partial p_2^*(t)}{\partial b_1(t)}$?
  $\Rightarrow$ Answer: it depends on α_3, the weight on the bequest function. (next page)

- Note that since α_3 is the weight on the bequest function, high (resp. low) α_3 implies strong (resp. weak) bequest motive.

Finding 2. Sign of $\dfrac{\partial p_1^*(t)}{\partial b_2(t)}$ and $\dfrac{\partial p_2^*(t)}{\partial b_1(t)}$

- If the bequest motive is weak (α_3 is less than some threshold), then $\frac{\partial p_1^*(t)}{\partial b_2(t)} < 0$. More precisely,

$$\frac{\partial p_1^*(t)}{\partial b_2(t)} < 0, \quad \text{if } \alpha_3 < \left\{ \frac{\gamma_3(\lambda_1 + K_3)(\lambda_1 + \lambda_2 + K_3)}{\gamma_1(\lambda_1 + K_1)\lambda_2 K_3} \right\} y^*(t)^{1-\frac{\gamma_3}{\gamma_1}} \alpha_1^{\frac{\gamma_3}{\gamma_1}}.$$

Meaning: If the bequest motive is weak, then it is optimal to reduce the life insurance on wage earner 1 as the expected future labor income of wage earner 2 increases.

Intuition: If the bequest motive is weak, then the main reason of paying life insurance premium on wage earner 1 is to guarantee enough wealth level for wage earner 2 after the death of wage earner 1. Thus, in this case, as $b_2(t)$ increases, i.e. the wage earner 2 becomes more financially independent, then less life insurance premium on wage earner 1 is enough for the family.

- In contrast, if the bequest motive is strong (α_3 is greater than some threshold), then $\frac{\partial p_1^*(t)}{\partial b_2(t)} > 0$. More precisely,

$$\frac{\partial p_1^*(t)}{\partial b_2(t)} > 0, \quad \text{if } \alpha_3 > \left\{ \frac{\gamma_3(\lambda_1 + K_3)(\lambda_1 + \lambda_2 + K_3)}{\gamma_1(\lambda_1 + K_1)\lambda_2 K_3} \right\} y^*(t)^{1-\frac{\gamma_3}{\gamma_1}} \alpha_1^{\frac{\gamma_3}{\gamma_1}}.$$

Meaning: Above result means that as the income of wage earner 2 increases, it is optimal for the family to buy more life insurance on the wage earner 1, as well as that on the wage earner 2 if the bequest motive is strong enough.

Intution: If the bequest motive is strong, then the optimal bequest level is high. Thus, increase of wage earner 2's income allows the family to pay more life insurance premium on wage earner 1, which results in higher insurance benefit at the death of wage earner 1, and thus higher wealth level after the death of wage earner 1.

- Similarly, the sign of $\frac{\partial p_2^*(t)}{\partial b_1(t)}$ is also different depending on the level of bequest motive (α_3) as follows:

$$\frac{\partial p_2^*(t)}{\partial b_1(t)} < 0, \quad \text{if } \alpha_3 < \left\{ \frac{\gamma_3(\lambda_2 + K_3)(\lambda_1 + \lambda_2 + K_3)}{\gamma_2(\lambda_2 + K_2)\lambda_1 K_3} \right\} y^*(t)^{1-\frac{\gamma_3}{\gamma_2}} \alpha_2^{\frac{\gamma_3}{\gamma_2}},$$

$$\frac{\partial p_2^*(t)}{\partial b_1(t)} > 0, \quad \text{if } \alpha_3 > \left\{ \frac{\gamma_3(\lambda_2 + K_3)(\lambda_1 + \lambda_2 + K_3)}{\gamma_2(\lambda_2 + K_2)\lambda_1 K_3} \right\} y^*(t)^{1-\frac{\gamma_3}{\gamma_2}} \alpha_2^{\frac{\gamma_3}{\gamma_2}}.$$

- Question: Until now, we have examined the signs of $\frac{\partial p_2^*(t)}{\partial b_1(t)}$ and $\frac{\partial p_1^*(t)}{\partial b_1(t)}$ separately. What about the sign of $\frac{\partial p_1^*(t)+p_2^*(t)}{\partial b_1(t)}$?

Finding 3. Sign of $\frac{\partial(p_1^*(t)+p_2^*(t))}{\partial b_1(t)}$ and $\frac{\partial(p_1^*(t)+p_2^*(t))}{\partial b_2(t)}$ if $b_1(t)+b_2(t)$ is fixed.

- $p_1^*(t)+p_2^*(t)$ can be written as

$$p_1^*(t)+p_2^*(t)=\lambda_1 b_1(t)+\lambda_2 b_2(t)+\Big(\text{function of } y^*(t)\Big). \tag{5}$$

- Recall that

$$X(t)+b_1(t)+b_2(t)=\alpha_1^{\frac{1}{\gamma_1}}\frac{1}{\lambda_1+K_1}y^*(t)^{-\frac{1}{\gamma_1}}+\alpha_2^{\frac{1}{\gamma_2}}\frac{1}{\lambda_2+K_2}y^*(t)^{-\frac{1}{\gamma_2}}$$
$$+\alpha_3^{\frac{1}{\gamma_3}}\left(\frac{\lambda_1\lambda_2}{\lambda_1+K_3}+\frac{\lambda_1\lambda_2}{\lambda_2+K_3}\right)\frac{1}{\lambda_1+\lambda_2+K_3}y^*(t)^{-\frac{1}{\gamma_3}}.$$

Thus, we can easily verify that if $X(t)$ is given and $b_1(t)+b_2(t)$ remains same, then $y^*(t)$ also remains same even if $b_1(t)$ and $b_2(t)$ change (as long as $b_1(t)+b_2(t)$ unchanged). Therefore, we can derive the following results in the next page.

- Depending on the mortality intensities, we have the following results:
- If $\lambda_1 > \lambda_2$, then

$$\frac{\partial(p_1^*(t) + p_2^*(t))}{\partial b_1(t)} > 0, \quad \frac{\partial(p_1^*(t) + p_2^*(t))}{\partial b_2(t)} < 0. \tag{6}$$

Intuition: In this case, if $b_1(t)$ increases (but $b_1(t) + b_2(t)$ is fixed), the human capital of the couple becomes more risky, while the total human capital of the couple does not change. Thus, the total life insurance premium increases.

- If $\lambda_1 < \lambda_2$, then it is just the opposite:

$$\frac{\partial(p_1^*(t) + p_2^*(t))}{\partial b_1(t)} < 0, \quad \frac{\partial(p_1^*(t) + p_2^*(t))}{\partial b_2(t)} > 0. \tag{7}$$

- If $\lambda_1 = \lambda_2$, the total life insurance is not affected by $b_1(t)$ as long as $b_1(t) + b_2(t)$ unchanged.

$$\frac{\partial(p_1^*(t) + p_2^*(t))}{\partial b_1(t)} = 0 \tag{8}$$

Finding 4. Sign of $\dfrac{\partial(p_1^*(t)+p_2^*(t))}{\partial b_1(t)}$ and $\dfrac{\partial(p_1^*(t)+p_2^*(t))}{\partial b_2(t)}$ without assuming fixed $b_1(t)+b_2(t)$

- In the previous result on the signs of $\dfrac{\partial(p_1^*(t)+p_2^*(t))}{\partial b_1(t)}$ and $\dfrac{\partial(p_1^*(t)+p_2^*(t))}{\partial b_2(t)}$ when $b_1(t)+b_2(t)$ is fixed, the strength of bequest motive (α_3) is irrelevant.
- However, if $b_1(t)+b_2(t)$ is not fixed, then α_3 matters.
- If $\alpha_3 = 0$, then we have a result coincides with the result when $b_1(t)+b_2(t)$ is fixed. (next page)

- If $\alpha_3 = 0$, then we have the following results:
  - If $\alpha_3 = 0$ and $\lambda_1 > \lambda_2$,

$$\frac{\partial(p_1^*(t) + p_2^*(t))}{\partial b_1(t)} > 0, \quad \frac{\partial(p_1^*(t) + p_2^*(t))}{\partial b_2(t)} < 0.$$

  - If $\alpha_3 = 0$ and $\lambda_1 = \lambda_2$,

$$\frac{\partial(p_1^*(t) + p_2^*(t))}{\partial b_1(t)} = 0, \quad \frac{\partial(p_1^*(t) + p_2^*(t))}{\partial b_2(t)} = 0,$$

  - If $\alpha_3 = 0$ and $\lambda_1 < \lambda_2$,

$$\frac{\partial(p_1^*(t) + p_2^*(t))}{\partial b_1(t)} < 0, \quad \frac{\partial(p_1^*(t) + p_2^*(t))}{\partial b_2(t)} > 0.$$

- However, if $\alpha_3 > 0$, it becomes a little bit more complicated (omitted).

Future Research

This is still an ongoing work, and we will work on the following:

- Connect our model to other empirical findings on family's life insurance demand.
- Generalize the setup (for example, time-varying mortality intensities, stochastic income, other utility functions, and so on...)

References

Beck, T., Webb, I., 2003. Economic, Demographic, and Institutional Determinants of Life Insurance Consumption across Countries. *The World Bank Economic Review* 17, 51–88.

Fitzgerald, J., 1987. The Effects of Social Security on Life Insurance Demand by Married Couple. *Journal of Risk and Insurance* 54, 86–99.

Hau, A., 2000. Liquidity, Estate Liquidation, Charitable Motives, and Life Insurance Demand by Retired Singles. *Journal of Risk and Insurance* 67, 123–141.

Liebenberg, A.P., Carson, J.M., Dumm, R.E., 2012. A Dynamic Analysis of the Demand for Life Insurance. *Journal of Risk and Insurance* 79, 619–644.

Nguyen, P., Li, D., Moshirian, F., 2007. The Demand for Life Insurance in OECD Countries. *Journal of Risk and Insurance* 74, 637–652.

Zietz, E.N., 2003. An Examination of the Demand for Life Insurance. *Risk Management and Insurance Review* 6, 159–191.