Robust dynamic pairs trading with cointegration

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Cointegration

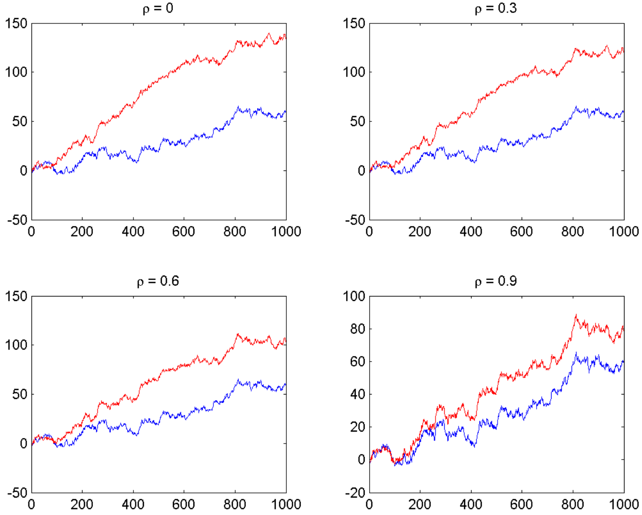
- Granger (1981, J. Econometrics): A linear combination of two or more non-stationary time series could be stationary.
- Engle and Granger (1987, Econometrica) denoted this property as cointegration: co-movements among trending variables.
- The concept of cointegration in financial time series leads to Nobel memorial prize in Economics for Granger in 2003.
- This work investigates optimal pairs-trading of cointegrated risky assets using utility maximization and robust portfolio rules.

Cointegration: Empirical evidence

- International stock market: Cerchi and Havenner (1988, JEDC), Taylor and Tonks (1989, Rev. Eco. Stat.)
- Exchange rates: Baillie and Bollerslev (1989, JF), Kellard et al. (2010, JBF)
- Crude oil, heating oil and gasoline futures: Serletis (1992, Energy Eco.)
- Oil spot and future prices: Maslyuka and Smyth (2009, JBF)

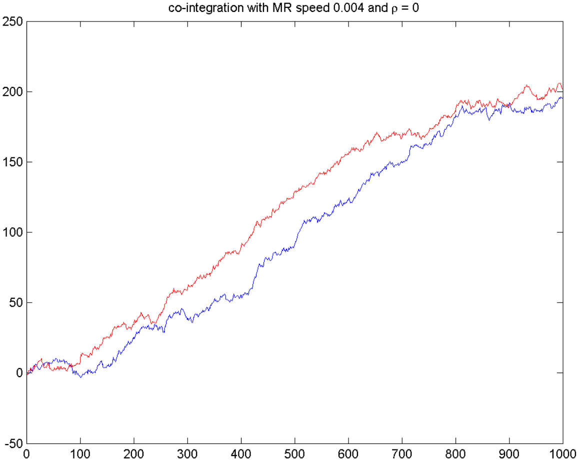
Contegration vs. Correlation

Figure 1: Pairs of Brownian motions with different correlations.



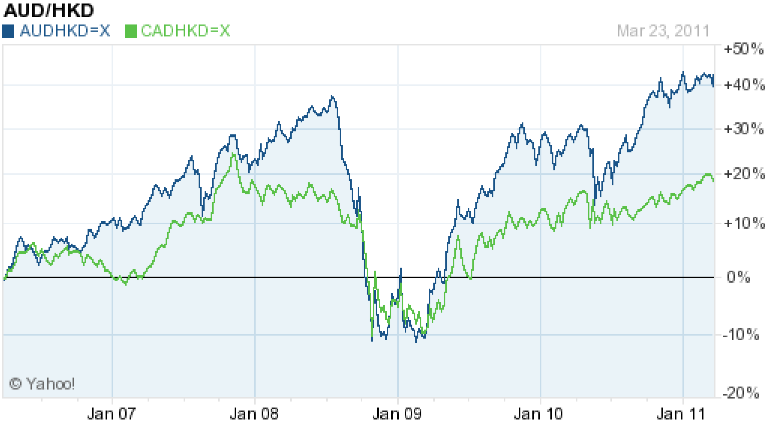
Cointegration vs. Correlation

Figure 2: A pair of cointegrated processes with $\rho = 0$.



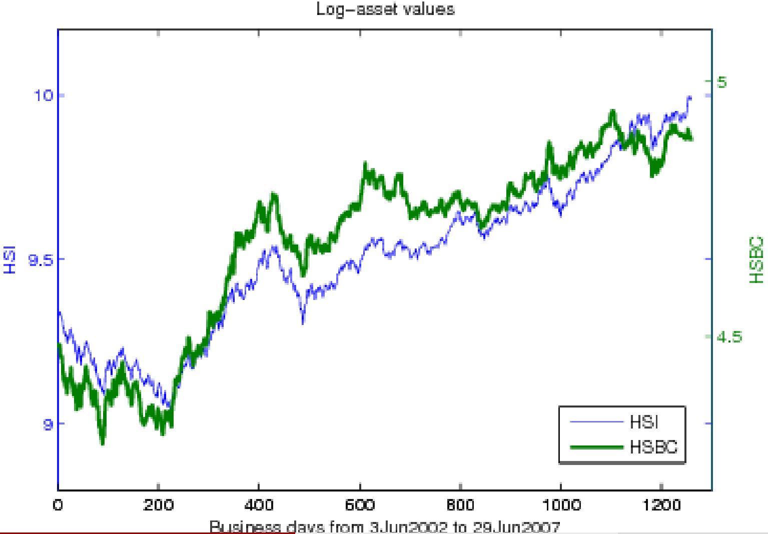
Cointegration vs. Correlation

Figure 3: FX Market. Correlation or cointegration?



Cointegration vs. Correlation

Figure 4: Stock Market. Correlation or cointegration?



Cointegration

- Index-tracking with cointegration: Alexander and Dimitriu (2005)
- Practical use of cointegration in pairs trading: Vidyamurthy (2004)
- Relative-value strategy: Gatev et al. (2006)
- MV pairs trading strategy: Chiu and Wong (2011 JEDC, 2012 EJOR)
- Constant relative risk aversion (CRRA) pairs trading strategy: Liu and Timmermann(2013 Rev. Fin. Studies)
- Constant absolute risk aversion (CARA) pairs trading strategy: Tourin and Yan (2013 JEDC)

Empirical Situation

- Lei and Xu (2015 JEDC) empirically show that the profit is much reduced for the existence of transaction cost.
- Another practical issue is the estimation errors of parameters in the cointegration model.
- The finance literature has reported the distortion of empirical optimal portfolio performance by estimation errors.

Robustness

- Knight (1921 Risk, Uncertainty and Profit) and Maenhout (2004 Rev. Fin. Studies): investment decision is made under the worst case among scenarios characterized by a set of equivalent probability measures.
- The classic expected utility maximization approach is replaced by a maximin problem.
- Such an approach has been applied to diffusion processes with deterministic drifts (Yi et al., 2013 IME; Pun and Wong, 2015 IME)

Cointegration Model I

The cointegration model in Engle and Granger (1987) will be used. In discrete-time case, the error correction dynamics with k $(1 \leq k \leq n)$ cointegrating factors:

$$\begin{aligned} \ln S_{i,t} - \ln S_{i,t-1} &= \mu + \sum_{j=1}^{k} \delta_{ij} z_{j,t-1} + \sigma_{i,t}\epsilon_{i,t}, i = 1, 2, \ldots, n, \quad (1) \\ z_{j,t} &= a_j + b_j t + \sum_{i=1}^{n} c_{ij} \ln S_{i,j}, for j = 1, \ldots, k, \end{aligned}$$

where the random vectors $[\epsilon_{1,t}, \ldots, \epsilon_{n,t}]$ follows multivariate normal with zero mean and constant correlation coefficient matrix.

Cointegration Model II

Consider the case with two assets and one cointegrating factors.
The cointegrating relationships (if z_t tested stationary):

$$z_t = a + bt + c \ln S_{2,t} - \ln S_{1,t}$$

where a, b, c can be estimated through ordinary least square method:

$$\ln S_{1,t} = a + bt + c \ln S_{2,t} + z_t$$

- Estimate of $\hat{c}$ converges with rate of $1/N$.
  Estimate of $\hat{a}$ and $\hat{b}$ converge with rate of $1/\sqrt{N}$ (Stock 1987).
- Estimation error of $\hat{b}$ determines the optimal asset allocation and is a more serious concern.

Continuous-time model I

In the continuous-time model, the cointegration model is the diffusion limit of the discrete-time ECM, which is derived to be (Duan and Pliska 2004):

$$\begin{aligned} d\ln S_{i,t} &= \left(\mu + \sum_{j=1}^{k} \delta_{ij} z_{j,t}\right) dt + \sigma_i d\widehat{W}_{i,t}, \quad i = 1, \ldots, n, \\ z_{j,t} &= a_j + b_j t + \sum_{i=1}^{n} c_{ij} \ln S_{i,t} \quad j = 1, \ldots, k, \end{aligned} \tag{2}$$

where $\widehat{W}_{i,t}$ are correlated Wiener processes.

Continuous-time model II

- Subsititute (3) into (2) and we have

$$d \ln S_t = (\theta - \mathcal{A} \cdot \ln S_t)\, dt + \sigma dW_t,$$

where $\theta \in \mathbb{R}^n$,
$W(t)$ is n-dimensional uncorrelated Wiener process,
$\mathcal{A}$ is $n \times n$ constant matrix of cointegration coefficients,
$\sigma(t)\sigma(t)'$ is the covariance matrix of assets.

Market

- A risk-free asset, $S_0(t)$: $dS_0(t) = r(t)S_0(t)dt, \quad S_0(0) = R_0 > 0$
- n risky assets with log-price process, $X_i(t) = \ln S_i(t)$, $i = 1, 2, \ldots, n$:

$$dX(t) = [\theta(t) - AX(t)]\, dt + \sigma(t)dW(t), t \in [0, T] \quad (3)$$

  which is the same as defined in the cointegration model.
- Also assume that the non-degeneracy condition of $\sigma(t)\sigma(t)' \succcurlyeq \delta I_n$ holds for all $t \in [0, T]$ and for some $\delta > 0$.

Robust Optimal Investment I

- Define Wealth Process $Y(t) = \sum_{i=0}^{n} u_i(t)$,
- $u_i(t)$: amount invested in asset i
- Admissible portfolio: $\mathrm{E}[\int_0^T \|u(\tau)\|^2 d\tau] < \infty$
- Apply Itô lemma to $Y(t)$ w.r.t. cointegrating dynamics

$$\begin{aligned} dY(t) &= \left[r(t)Y(t) + u(t)'B(t)\right] dt + u(t)'\sigma(t)dW_t, \\ Y(0) &= Y_0, \end{aligned}$$

where

$$B(t) = \theta - AX(t) + \frac{1}{2}\mathcal{D}(\sigma(t)\sigma(t)')\mathbf{1} - r(t)\mathbf{1}, \tag{4}$$

and $\mathcal{D}(\sigma(t)\sigma(t)')$ is the diagonal matrix with all diagonal elements equal to those of $\sigma(t)\sigma(t)'$

Robust Optimal Investment II

- Classical utility maximization problem:

$$\max_{u(\cdot)} \quad \mathsf{E}^{\mathcal{P}}\left[U(Y(T))\right] \text{ s.t. } (3),(4) \text{ and } u(\cdot) \in \Pi,$$

for all admissible portfolio $u(\cdot)$ and the utility of an investor $U(\cdot)$.

Alternative robust model:

- Consider a class of equivalent probability measure $\mathfrak{Q} := \{\mathcal{Q}|\mathcal{Q} \sim \mathcal{P}\}$ there exists a well-defined stochastic process $\varphi^{\mathcal{Q}}$ for any $\mathcal{Q} \in \mathfrak{Q}$

$$\frac{d\mathcal{Q}}{d\mathcal{P}} = \exp\left(\int_0^t \varphi^{\mathcal{Q}}(s)' dW_s - \frac{1}{2}\int_0^t \left\|\varphi^{\mathcal{Q}}(s)\right\|^2 ds\right),$$

and $\varphi^{\mathcal{Q}}(s)$ satisfies the Novikov condition.

Robust Optimal Investment III

Adopting Maenhout (2004) and Ye at al. (2013) notion of robustness optimization, the objective function with ambiguity aversion reads:

$$\sup_{u\in\Pi}\inf_{\mathcal{Q}\in\mathfrak{Q}} \mathsf{E}^{\mathcal{Q}}\left[U(Y(T))+\frac{1}{2\xi}\int_0^T \frac{\|\varphi^{\mathcal{Q}}(s)\|^2}{\phi(s)}ds\right], \tag{5}$$

where ξ is a measure of ambiguity aversion,
$\frac{\|\varphi^{\mathcal{Q}}(s)\|^2}{2}$ measures the relative entropy between $\mathcal{P}$ and $\mathcal{Q}$,
and $\phi(t)\geq 0$ is the preference parameter related to ambiguity aversion.

Robust Optimal Investment IV

- $\frac{1}{2}\int_0^T \frac{\|\varphi^{\mathcal{Q}}(s)\|^2}{\phi(s)}ds$ means the penalty for the model choice in accordance with the preference parameter. (Maenhout 2004)
- If $\xi \to 0$, the problem is reduced to the classical utility maximization problem with cointegrated assets.
  If $\xi \to \infty$, the penalty function vanishes and all candidate measures are identical. (Zhang and Siu 2009)

Explicit Solution for CARA Investors I

In our research problem, the value function under CARA utility objectives:

$$\sup_{u\in\Pi}\inf_{\mathcal{Q}\in\mathfrak{Q}} \mathsf{E}^{\mathcal{Q}}\left[1-e^{-\lambda Y(T)}-\frac{1}{2\xi}\int_t^T \frac{(V_y(s,y,\beta))^2\,\|\varphi^{\mathcal{Q}}(s,y,\beta)\|^2}{V_{yy}(s,y,\beta)}ds\,\middle|\,Y(0)=y_0,B(0)=\beta_0\right],$$

Based on the value function:

$$V(t,y,\beta)=\sup_{u\in\Pi}\inf_{\mathcal{Q}\in\mathfrak{Q}} \mathsf{E}^{\mathcal{Q}}\left[U(Y(T))+\frac{1}{2\xi}\int_t^T \frac{\|\varphi^{\mathcal{Q}}(s,y,x)\|^2}{\phi(s,y,x)}ds\,\middle|\,Y(t)=y,B(t)=\beta\right].$$

with the choice of $\phi(t,y,\beta)=\frac{1}{R(t,y,\beta)V_y},\ =-\frac{V_{yy}(t,y,\beta)}{\left(V_y(t,y,\beta)\right)^2}$. (Maenhout 2004)
and $U(y)=1-e^{-\lambda y},\quad \lambda>0$.

Explicit Solution for CARA Investors II

Solution:

$$u^*(t, B(t)) = \frac{1}{\lambda e^{\int_t^T r(s)ds}(1+\xi)} \left(\Sigma^{-1}(t)B(t) - (1+\xi)A'(K(t)B(t) + N(t)\right),$$

and the value function:

$$1 - \exp\left[\frac{1}{2}\beta' K(t,T)\beta + N'(t,T)\beta - \lambda y e^{\int_t^T r(s)ds} + \int_t^T \left(N'(s,T)\Theta(s) + \frac{1}{2}tr\left(K(s,T)A\Sigma(s)A'\right)\right) ds\right].$$

Explicit Solution for CARA Investors III

where

$$\begin{aligned} B(t) &= \theta - AX(t) + \frac{1}{2}\mathcal{D}(\sigma(t)\sigma(t)')\mathbf{1} - r(t)\mathbf{1} \\ K(t,T) &= -\frac{1}{1+\xi}\int_t^T \Sigma^{-1}(s)ds \\ N(t,T) &= -\frac{1}{1+\xi}\int_t^T \int_s^T \Sigma^{-1}(\tau)d\tau\ \Theta(s)ds; \end{aligned}$$

and

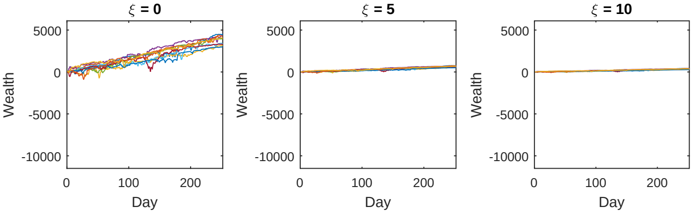
$$\Theta(t) = \dot{\theta}(t) + \frac{1}{2}\dot{\mathcal{D}}(t)\mathbf{1} - \dot{r}(t)\mathbf{1} + A\left(\frac{1}{2}\mathcal{D}\mathbf{1} - r\mathbf{1}\right)$$

Simulation I

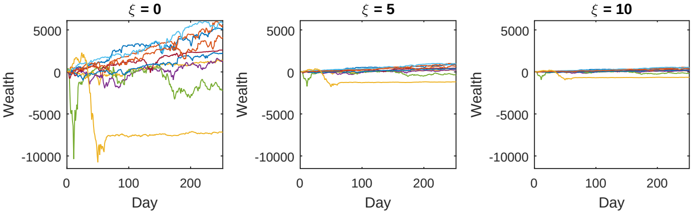
- Assumption: 250 log stock prices per year.
- $$X(t+\Delta t) - X(t) = (\theta - \mathcal{A}X(t))\Delta t + \sigma\sqrt{\Delta t}\epsilon$$
- where $$X(t) = \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} \text{ with initial values } X(0) = \begin{bmatrix} \ln 1 \\ \ln 2 \end{bmatrix},$$ $$\theta = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathcal{A} = \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix}, \sigma = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$ $$\epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} \text{ and } \epsilon_1, \epsilon_2 \sim N(0,1) \text{ i.i.d. }, \Delta t = \frac{1}{250}$$
- $\lambda = 0.5$ and r=0.03% p.a.
- The value of ξ was set from 0 to 10 (11 integers in total)

Simulation II

Results of 10 out of 100 simulations (Orcale Strategy)

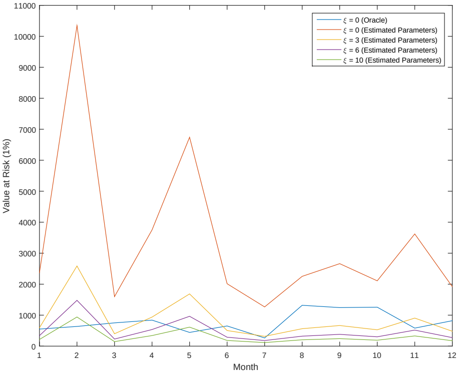


Results of 10 out of 100 simulations (Estimated Parameters)



Simulation III

The effect of robust approach in terms of VaR



Empirical Study I

- Targets: General Motors and Ford Motor
- Sampling period: 19/11/2010 to 31/8/2015 (1202 data points)
- Investment starts from 2/9/2014 to 31/8/2015
- Parameters estimated with expanding windows (updated monthly)
- ξ is selected based on maximizing the averaged monthly utility in previous periods

Empirical Study II

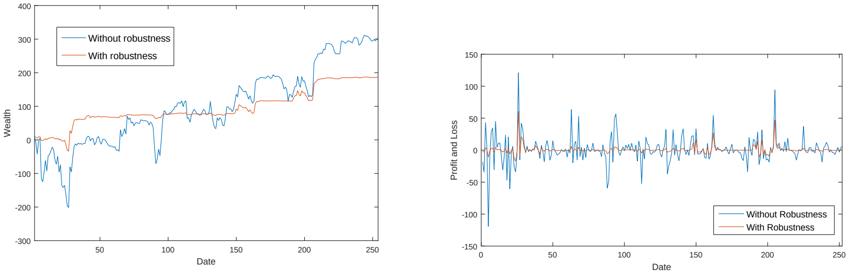


Figure: The wealth process of robust vs. classical pairs trading

Conclusion

- Classical approach of pairs trading with cointegration suffers from parameter estimation risk.
- Robust approach can reduce the impact of parameter estiamtion.
- A closed-form solution is obtained for the robust pairs trading strategy for CARA investors.
- We demonstrate that robust approach can reduce likelihood of extreme loss and is more stable than the classical counterpart.