Replication Pricing of xVA and the Asset-Liability Symmetry

Lixin Wu

Department of Mathematics
The Hong Kong University of Science and Technology

Joint work with Dawei Zhang, HKUST

Outline of the presentation

1. xVA — the new paradigm of derivatives pricing
2. Risk-neutral valuation and the bilateral replication strategy
3. The rise of other xVA
4. Market funding liquidity risk premium and asset-liability symmetry

Table of Contents

Option Pricing from 1973 $\sim$ 2000

Risk-neutral pricing

$$V_e(0) = E_0^Q[e^{-\int_0^T r_s ds} V_T],$$

where

r_t — the risk-free rate,
$\mathbb{Q}$ — A risk-neutral measure.

When the counterparty default risk is taken into account, the value becomes

$$V(0) = V_e(0) + CVA,$$

where CVA, for *credit valuation adjustment*, is defined by

$$CVA = -\text{Prob}(\tau \leq T) \times \underbrace{L}_{\text{Loss rate}} \times V_e(0).$$

- FASB 157 and IAS 39 were introduced in 2004 to price in counterparty risks.
- Banks started to make CVA on a monthly basis.

When the funding risk is further taken into account, then

$$V(0) = V_e(0) + CVA + DVA + FVA,$$

where
DVA — counterparty's CVA and
FVA — is for funding valuation adjustment , with a popular definition

$$FVA = -E_0^Q\left[\int_0^{\tau\wedge T} xe^{-\int_0^u r_s ds} V_e(u)du\right],$$

with x is the investor's funding spread. This formula is

- originated from hedging OTC swap with a fully collateralized swap.
- CVA and FVA formula have been generalized to a portfolio.

Definition of FVA

FVA — the funding cost/benefit resulted from borrowing or lending the shortfall/excess of cash arising from day-to-day derivatives business operations (for example, hedging and collateral posting/ receiving).

Risk Dynamics Blog, 2012

Triad of VAs

- A trade is made at the price $V_e(0)$.
- Yet the the trade can be booked using the economic value:

$$V(0) = V_e(0) + CVA + DVA + FVA.$$

- Making BCVA (i.e., CVA+DVA) is now a standard market practice.
- Some banks have made a triad of valuation adjustments (CVA, DVA and FVA), as early as 2012.
  - Royal Bank of Scotland reported FVA of £475 million (4Q 2012).
  - JP Morgan books $1.5bn FVA loss in the 4Q 2013 (Reuter).
  - Nomura books ¥10 billion FVA loss in the 4Q 2103 (Risk.net).
  - Citi takes $474 million FVA charge in 3Q 2014 (Risk.net).
  - Morgan Stanley takes $468m FVA loss in the 4Q 2014 (Risk.net).

The xVA

xVA means a list of valuation adjustments:

- Credit valuation adjustment (CVA) (settled).
- Funding valuation adjustment (FVA):
  - Funding cost adjustment (FCA).
  - Collateral valuation adjustment (COLVA).
  - Capital valuation adjustment (KVA).
  - Margin valuation adjustment (MVA).
  - Liquidity valuation adjustment (LVA).
- Taxation valuation adjustment (TVA).
- etc.

Discussions are undergoing on setting up the “xVA" desks (Carver, 2013).

Controversies surrounding the xVA

- Pricing in funding cost or booking FVA for P&L accounting cause asset-liability asymmetry.
- Hull and White (2012-) reject FVA.
- Over two-thirds say the current FVA approach is wrong - Risk.net poll (March 2015).
- The prevalent FVA models overstate costs (Albanese & Andersen, 2015).
- “There’s no convergence on how FVAs should be calculated, and everybody is doing it differently, because at the moment there are no right or wrong answers. Quite frankly, it’s a bit of a mess."
  — An auditor (Cameron, 2013)

xVA as parts of fair value

In literature xVA is often treated as part of “value to me".

- The master formula based on the risk-neutral valuation principle (Brigo *et al*., 2007-).
- Extending the Black-Scholes model for FVA (Piterbarg, 2010; Lou, 2015).
- BSDE approach (Crepey, 2011-12, Bichuch *et al*., 2015).
- Replication pricing of CVA and FVA (Burgard and Kjaer, 2011-);
- Enhanced replication pricing (Wu, 2013-).
- Replication pricing to include KVA and MVA (Green & Kenyon, 2014-15).

The main questions

For a derivative trade, bilateral or CCP,

- what should be the fair price?
- how should the trade be booked?

Our answers

- The fair price is $(NDV + BCVA) + FVA_{mkt}$.
- This price is for both trading and booking.

Asset-liability symmetry is retained in both trading and booking, and it is in line with the emerging market consensus!

Table of Contents

Some notations

Y_T	—	the contractual payoff of the derivative at maturity T
τ_i	—	the default time of party i, $i = B$ and C
$I_i(t)$	—	≥ 0, the value of the initial margin (IM)
$X_i(t)$	—	≥ 0, the value of the variable margin (VM) or collaterals
$K_i(t)$	—	≥ 0, the value of the capital
$V(t)$	—	the value of the derivative fair to B, the bank, s.t.
		$V(t) > 0$ — asset,
		$V(t) < 0$ — liability

Mathematical Setup

We will model the market by $(\Omega, \mathcal{G}, \mathcal{G}_t, \mathbb{P})$, where

$\mathbb{P}$ — the real-world measure,

$\mathcal{G}_t$ — $\mathcal{F}_t \vee \mathcal{H}_t$, where

$\mathcal{F}_t$: all market information except defaults, while

$\mathcal{H}_t$: default status, $\sigma(\{\tau_B \leq u\} \vee \{\tau_C \leq u\} : u \leq t)$

Minor assumptions:

1. the risk-free rate, CDS rates and recovery rates are constants;
2. there is no default by the current moment $t = 0$, i.e., $\tau > 0$.

The derivative payoff Y_T can be $\mathcal{G}_T$-adaptive (so there is no restriction on issuer of the underlying).

Features of the pricing problem

- An equity derivative.
- Counterparties default risks.
- Funding costs/risks
  - repo spreads (haircut for capital or fee for short selling);
  - borrowing spreads for IM, VM and capital;
  - rehypothecation benefit.

Pricing strategy

Two-step pricing strategy

1. Ignore funding costs, and
   - let the party of liability replicate the derivatives payoff (using e.g. repos);
   - let the party of exposure replicate the LGD (using CDS);
   - set the derivatives value as the difference of the replication costs.
2. Take into account funding costs, and calculate the xVA for replication, margin/collateral posting and capital.

Primary securities

The primary securities are saving account, share and CDS, which have the following $\mathbb{P}$ dynamics:

$$
\begin{aligned}
dB_t &= r_t B_t dt, \\
dS_t &= S_t \left[\mu_t dt + \sigma_t dW_t^{(P)} \right], \\
dU_i(t) &= r_t U_i(t) dt - s_i dt + L_i dJ_i^{(P)}, \quad i = B \text{ and } C,
\end{aligned}
$$

At hedge revision, we will use par CDS, such that $U_i(t) = 0$.

The risk-neutral measure

The risk-neutral measure is defined by

$$
\left.\frac{d\mathbb{Q}}{d\mathbb{P}}\right|_{\mathcal{F}_t} = \frac{e^{-\int_0^t \gamma_S(u)dW_u^{(P)}}}{E_0^P\left[e^{-\int_0^t \gamma_S(u)dW_u^{(P)}}\right]} \frac{e^{\gamma_B J_B^{(P)}(t)}}{E_0^P\left[e^{\gamma_B J_B^{(P)}(t)}\right]} \frac{e^{\gamma_C J_C^{(P)}(t)}}{E_0^P\left[e^{\gamma_C J_C^{(P)}(t)}\right]}
$$

$$
= e^{\int_0^t -\frac{1}{2}\gamma_S^2(u)du + \gamma_S(u)dW_u^{(P)} + \gamma_B J_B^{(P)}(t) + \lambda_B^{(P)} t(1-e^{\gamma_B}) + \gamma_C J_C^{(P)}(t) + \lambda_C^{(P)} t(1-e^{\gamma_C})},
$$

with

$$
\gamma_S(t) = \frac{\mu_t - r_t - \lambda_S + q_t}{\sigma_S},
$$

$$
\gamma_B = \ln\frac{s_B/L_B}{\lambda_B^{(P)}} \quad \text{and} \quad \gamma_C = \ln\frac{s_C/L_C}{\lambda_C^{(P)}}.
$$

The risk-neutral valuation

Proposition

The risk-neutral valuation of the derivative to the counterparties is

$$V_f(0) = E_0^Q \left[e^{-\int_0^{\tau \wedge T} r_s ds} V(\tau \wedge T) \right] \quad \square \tag{1}$$

BCVA as part of the fair price

Corollary

When Y_T is $\mathcal{F}_T$-adaptive, the risk-neutral valuation of the derivative has the following decomposition:

$$V_f(0) = V_e(0) + CVA_B + CVA_C, \tag{2}$$

where

$$\begin{aligned} CVA_B &= E_0^Q[1_{\{\tau=\tau_B\le T\}} e^{-\int_0^{\tau_B} r_s ds}(V(\tau_B) - V_e(\tau_B))], \\ CVA_C &= E_0^Q[1_{\{\tau=\tau_C\le T\}} e^{-\int_0^{\tau_C} r_s ds}(V(\tau_C) - V_e(\tau_C))] \quad \square \end{aligned} \tag{3}$$

Note that

- $V_e(0) + CVA_C$ is the replication cost of B, and
- CVA_B is the replication cost of C.

Table of Contents

The rise of other xVA

Once B and C enter into a trade at time $t = 0$ for a value V_0 to B, the two parties start hedging, posting margins/collaterals and setting aside capital. The hedging portfolios of the two parties can be expressed as

$$\begin{aligned} \Pi_B(t) &= \delta_B Z_S(t) + \alpha_B U_C(t) + \beta_B(t), \\ \Pi_C(t) &= \delta_C Z_S(t) + \alpha_C U_B(t) + \beta_C(t), \end{aligned} \tag{4}$$

where δ_i is the number of repo contracts held by party i, which can be

$$\delta_B = -\frac{\partial V_e(t)}{\partial S}, \quad \delta_C = \frac{\partial V_e(t)}{\partial S},$$

and α_i is the numbers of CDS contract taken up by party i to hedge against LGD:

$$\alpha_B(t) = -\frac{V(\tau_C = t) - V_e(t)}{L_C}, \quad \alpha_C(t) = -\frac{V(\tau_B = t) - V_e(t)}{L_B},$$

The cash accounts

$\beta_i(t)$ is the total value of party i's cash and funding transactions, such that

$$\beta_i(t) = \beta_{m,i}(t) + Z_i^{(I)}(t) + Z_i^{(X)}(t) + Z_i^{(K)}(t), \quad i = B \text{ and } C.$$

Here,

$\beta_{m,i}(t)$	—	saving account used for unsecured borrowing and for depositing the P&L from hedging and funding,
$Z_i^{(I)}(t)$	—	$= 0$, zero-net funding transactions for IM at time t,
$Z_i^{(X)}(t)$	—	$= 0$, zero-net funding transactions for VM or collateral at time t,
$Z_i^{(K)}(t)$	—	$= -\phi_i(t)K_i(t)$, value of funding transactions for capital, where $\phi_i(t) \in [0, 1]$ is the fraction of risk capital allocated t the saving account for funding purpose.

The cash accounts, cont'd

Initially, the balances of the parties' saving accounts are

$$\beta_{m,B}(0) = -V_0 + \phi_B(0)K_B(0), \quad \beta_{m,C}(0) = V_0 + \phi_C(0)K_C(0),$$

and the initial values of the hedging portfolios are

$$\Pi_B(0) = -V_0, \quad \text{and} \quad \Pi_C(0) = V_0.$$

Here, V_0 is the initial premium payment paid by or received by B, depending on its being positive or negative.

The rise of the funding costs

the total value of cash of party B evolves according to

$$\begin{aligned} d\beta_B(t) &= d\beta_{m,B}(t) + dZ_B^{(I)}(t) + dZ_B^{(X)}(t) + dZ_B^{(K)}(t) \\ &= \Big((r_t + x_B)\beta_{m,B}(t) - x_B^{(I)} I_B(t) - x_B^{(X)} X_B(t) - \gamma_B^{(K)} K_B(t)\Big)\, dt, \end{aligned} \quad (5)$$

which is subject to initial balance $\beta_B(0) = -V_0$ due to making the premium payment (or taking the payment, if positive).

The saving account

While $I_B(t), X_B(t)$ and $K_B(t)$ are specified exogenously, $\beta_{m,B}(t)$ evolves according to

$$\begin{aligned} d\beta_{m,B}(t) = & \left((r_t + x_B)\beta_{m,B}(t) - x_B^{(I)} I_B(t) - x_B^{(X)} X_B(t) - \gamma_B^{(K)} K_B(t)\right) dt \\ & + d[\phi_B K_B] - \partial_S V_e(t) dZ_S(t) + \lambda_C^{(Q)}[V(\tau_C = t) - V_e(t)]dt, \\ \beta_{m,B}(0) = & - V_0 + \phi_B K_B(0), \end{aligned} \tag{6}$$

so that the saving account that takes the P&L from hedge, margin/ collateral revisions and capital revisions.

Definition of the FVA

Definition

Funding valuation adjustment (FVA) to party B is the expected value of excess cost due to the funding spreads for various funding transactions:

$$FVA_B = E_0^Q\left[e^{-\int_0^{\tau\wedge T} r_s ds}\beta_B(\tau\wedge T)\right] - \beta_B(0).$$

This definition tolerates the mispricing of derivatives.

Valuation of the FVA

Proposition

To party B, the funding valuation adjustment is given by

$$FVA_B = FCA_B + MVA_B + ColVA_B + KVA_B, \tag{7}$$

Valuation of the xVA

Proposition (cont'd)

where

$$
\begin{aligned}
FCA_B =& E_0^Q\left[\int_0^{T\wedge\tau} x_B e^{-\int_0^t r_s ds}\beta_{m,B}(t)dt\right],\\
MVA_B =& -E_0^Q\left[\int_0^{T\wedge\tau} x_B^{(I)} e^{-\int_0^t r_s ds} I_B(t)dt\right],\\
ColVA_B =& -E_0^Q\left[\int_0^{T\wedge\tau} x_B^{(X)} e^{-\int_0^t r_s ds} X_B(t)dt\right],\\
KVA_B =& -E_0^Q\left[\int_0^{T\wedge\tau} \gamma_B^{(K)} e^{-\int_0^t r_s ds} K_B(t)dt\right],
\end{aligned}
\tag{8}
$$

and $\beta_{m,B}(t)$ *evolving according to (6)* □

Some comments

- It is the most general results so far.
  - It is valid for incomplete markets.
  - It includes those of Burgard and Kjaer (2012), Green and Kenyon (2015), etc.
  - Can be adapted to OTC (CSA or No-CSA) or CCR trades.
- The MtM value for calculating LGD can only be $\hat{V}_e(t)$.

“The value to me"

- The value to a trader is the value with which the risk-neutral return is the risk-free rate.
- Let $V_0^{(i)}$ be the value to party i, then (Li and Wu, 2015)

$$\begin{aligned} V_0^{(B)} &= V_f(0) + FVA_B(-V_0^{(B)}), \\ -V_0^{(C)} &= -V_f(0) + FVA_C(V_0^{(C)}), \end{aligned} \tag{9}$$

  which are nonlinear in general.
- In general,

$$V_0^{(B)} \neq V_0^{(C)}!$$

Table of Contents

Market funding liquidity risk premium

Proposition

Assume independence between market funding risk and counterparty default risks. When all but the systematic (or market) funding risk premium, x_m, are ignored, the bid and ask prices of a derivative are identical and are given by

$$V_0^{(B)} = \frac{V_f(0)}{1 + x_m E_0^Q[\tau \wedge T]} = V_0^{(C)} \quad \square$$

Risk management and accounting treatments of FVA

- In risk management
  - Deduct FVA from CET1 capital (Albabese *et al.*, 2015); or
  - We propose the *VaR* of *CVaR* measure for funding risks, under the $\mathbb{P}$ measure.
    - Stress testing funding risks.
    - Set the duration equal to the reporting period.
- In accounting
  - We should consider using the *realized funding costs*,

$$FC_B(t) = \int_0^t g_B(u)du, \tag{10}$$

for P&L attribution.

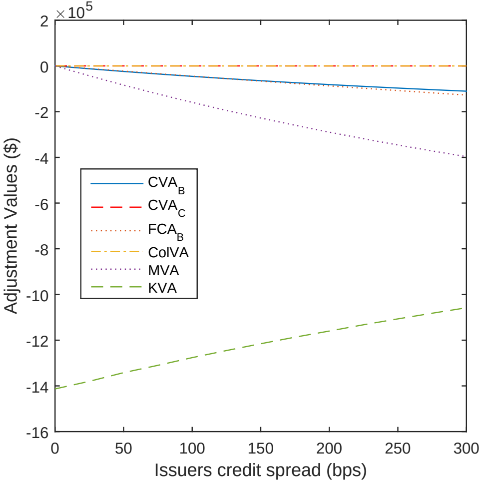

Figure 1. The xVA for the 20-year ATM swap.

Thank You!

www.math.ust.hk/˜malwu