

Informed Traders' Hedging with News Arrivals

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Outline

- 1 Introduction
- 2 Previous Studies
- 3 Basics
- 4 Minimal Martingale Measure
- 5 Local Risk Minimization Strategy
- 6 What to do next?

Information Asymmetry

Background

- In a market, different traders have **different levels of information**.
- Even when two traders have the exactly same information, they may interpret the information in different ways, or make different decisions.
- **Information is modeled by a filtration** in mathematical finance theory.
- A trader with more information has a larger filtration than a trader with less information.

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- **Insider**(informed trader): a trader with more (exclusive) information or better interpretation skill of the public information.
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How to model?

- We introduce an **information process**.
- This exclusive information often causes bigger movements than those usual diffusion can explain, and it is natural to involve this information to **jump** terms.
- jump in the price process itself? jump in the volatility term? jump size? jump timing(intensity)?

Filtration

- Let \mathcal{F} be the filtration generated by the market. It is an honest trader's filtration.
- An insider has a larger filtration \mathcal{G} available only to insiders.
- $\mathcal{F} \subset \mathcal{G}$.
- Kyle(1985), Amendinger(2000), Biagini and Oksendal(2005) assumed that the $\mathcal{G}_t = \mathcal{F}_t \vee \sigma(L)$ for some fixed random variable L . (usually a future price)
- Hu and Oksendal(.) studied a model that more and more additional information is available to the investor as time goes by. They used a sequence of random variables available only to insiders as additional information at certain points of times.(scheduled announcements)
- We generalize these studies to the case with $\mathcal{G}_t = \mathcal{F}_t \vee \sigma(X_s, 0 \leq s \leq t)$, where the additional information X given to insiders is not a single random variable nor a discrete sequence of random variables, but a diffusion process.

Research on Information Effects

Q: Obviously, an informed trader should do better in the market. But how can we mathematically explain and support this? More specifically, how can we find an optimal hedging strategy and pricing for an informed trader?

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Earning announcement and earning jumps

- Lee and Leung
- deterministic time jump
- learning procedure → Brownian bridge

Research on Information Effects

Q: Other information issues

more finance* papers

- K.Lee, R.Christie-David, A. Chatrath and B.Adrangi(Journal of Futures Markets, Volume 31, Issue 10, pages 915-946, October 2011)
 - Dominant markets, staggered openings, and price discovery
 - [Spillover effect, leading-following interaction](#)
- K.Lee, R.Christie-David and A. Chatrath(Journal of Futures Markets, vol 29, (1), 42-73, 2009):
 - How potent are news reversals?: Evident from futures markets
 - [Surprise!](#)

Lee and Song(2007)

$$dS_t = f(S_{t-})dB_t + g(S_{t-})dR_t + h(S_{t-})dt \quad (1)$$

$$0 \leq t \leq T$$

where

- $R_t = \sum_{n=1}^{N_t} U_n$
- $N_t - \int_0^t \lambda(X_s)ds =$ a local martingale under \mathbf{P}
- X , which is a firm specific information available only to insiders, satisfies the stochastic differential equation $dX_t = \alpha(X_t)dt + \beta(X_t)dB_t^X$ for $0 \leq t \leq T$.
- B' is another standard Brownian motion under \mathbf{P} such that $[B, B^X]_t = \rho t$.
- Correlation ρ between two Brownian motions B and B^X explains the level of exclusive information.
- U_n is i.i.d and has a pdf ν on $(-1, 1)$
- U_n denotes the jump sizes of S_t and has mean 0 and a finite second moment σ^2 .

Kang and Lee(2014)

$$dS_t = S_{t-}(\mu dt + \sigma dB_t + dR_t), 0 \leq t \leq T \quad (2)$$

where

- B_t is a standard Brownian motion.

-

$$R_t = \sum_{0 < s \leq t} \theta(X_s) 1(\Delta N_s = 1)$$

where $\theta(\cdot)$ is an increasing function and $-1 < \theta(x) < \frac{\sigma^2}{\mu}$.

- N_t is a Poisson counting process with rate λ under \mathbf{P} . $\hat{N}_t := N_t - \lambda t$ is a martingale under \mathbf{P} .

-

$$dX_t = \alpha(X_t)dt + \beta(X_t)dB_t^X, \quad X_0 = x_0.$$

where B^X is a standard Brownian motion with $[B, B^X]_t = \rho t$.

Kang and Lee(2014)

Assuming that honest traders believe the Black-Scholes model,

Table: $E[(C_T - C_0)^2]$, $\rho = -0.5$

Vol	10%	20%	30%
Vol Ratio	1.934157	1.299467	1.154251
80	0.104438, 0.751323	1.028191, 1.189337	1.851046, 1.792153
100	2.036793, 4.945836	1.537828, 1.686250	4.415904, 4.045074
120	0.721526, 0.922125	1.702788, 2.112400	2.645012, 1.369176

Table: $E[(C_T - C_0)^2]$, $\rho = 0.0$

Vol	10%	20%	30%
Vol Ratio	1.988265	1.317293	1.148387
80	0.080125, 1.069857	0.269744, 0.830191	0.962809, 1.119722
100	1.568441, 5.419202	1.047886, 1.557627	1.606752, 1.889270
120	0.693646, 2.347789	1.366413, 2.494248	1.683573, 1.805261

Kang and Lee(2014)

Table: $E[(C_T - C_0)^2]$, $\rho = 0.5$

Vol	10%	20%	30%
Vol Ratio	1.981465	1.318082	1.157330
80	0.611927, 1.590476	0.289834, 0.699058	0.814961, 1.101575
100	0.397466, 2.538860	1.052195, 1.639205	1.729693, 1.940148
120	1.072975, 1.680556	1.362013, 1.727360	1.875722, 1.793749

Park and Lee(2016)

- Informed traders in multiple levels \rightarrow level k trader

Model

$$dS_t = \mu S_t dt + f(\mathbf{Y}_t) S_t dW_t^{(0)}, \quad (3)$$

$$dY_t^{(i)} = \alpha_i(t, Y_t^{(i)}) dt + \beta_i(t, Y_t^{(i)}) dW_t^{(i)} + \gamma_i(t, Y_t^{(i)}) dR_t^{(i)}, \quad i = 1, \dots, n. \quad (4)$$

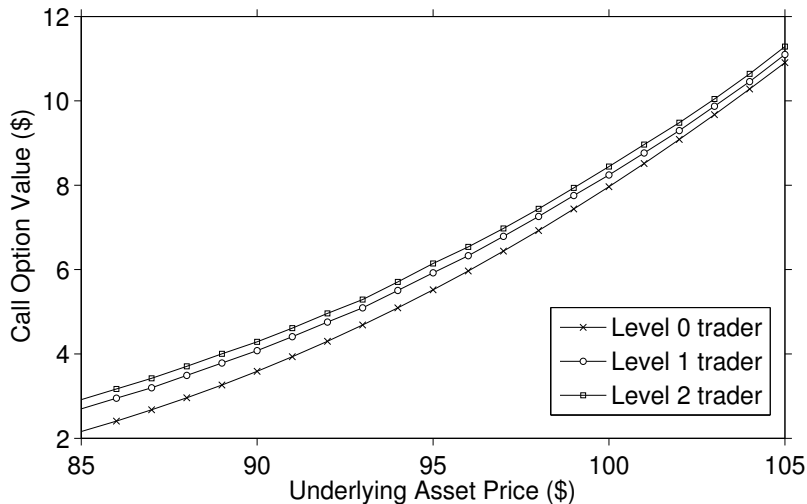
on a $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0 \leq t \leq T}, \mathbf{P})$ where \mathbf{P} is the empirical probability measure, and $\mathbf{Y} = (Y^{(1)}, \dots, Y^{(n)})$.

- $R_t^{(i)} = \sum_{j=1}^{N_t^{(i)}} U_j^{(i)}$.
- $U_j^{(i)}$: i.i.d. random variables with densities ν_i ,
- $E[U_j^{(i)}] = 0$ and $E[|U_j^{(i)}|^2] = \eta_i^2$.
- $N^{(i)}$: a Poisson process with bounded intensity λ_i .
- ρ_{ij} : correlation between $W^{(i)}$ and $W^{(j)}$

- different types of information: scheduled, randomly arriving, continuous etc.

Park and Lee(2016)

- How much improvement by additional information?



Park and Lee(2016)

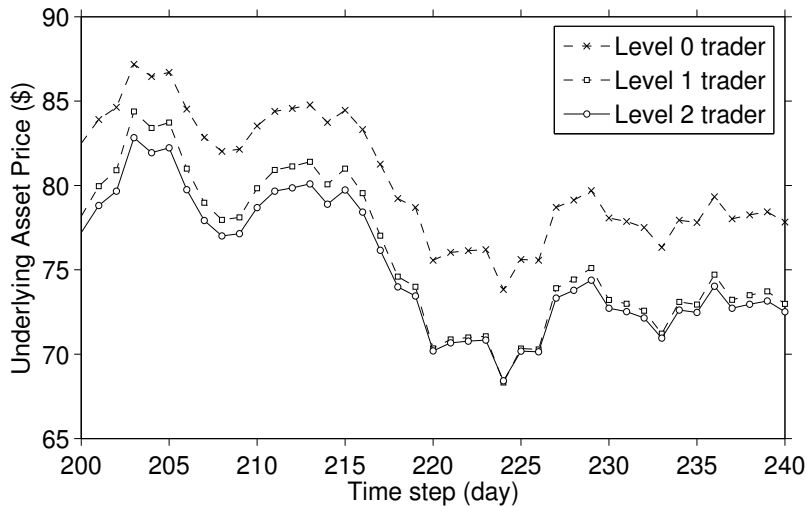


Figure: Sample Path of the Underlying

Basics

- We consider a market with **one risky asset** (S_t) and **one riskless asset** which would be assumed 1.
- **Portfolio**: a pair of processes (ξ_t, η_t) , $V_t = \xi_t S_t + \eta_t$
- **Contingent claim**: $H = H(S_T)$ at time T .
- **Cost process** of a portfolio (ξ_t, η_t) : $C_t = V_t - \int_0^t \xi_u dS_u$, $0 \leq t \leq T$

Hedging(replicating)

A (perfect) hedging portfolio(strategy) for a contingent claim $H(S_T)$ should satisfy the following two conditions.

- 1 Self-financing:

$$V_t = \xi_t S_t + \eta_t = \xi_0 S_0 + \eta_0 + \int_0^t \xi_u dS_u$$

- 2 Perfect match at maturity: $H(S_T) = V_T$

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- For a self financing portfolio, the cost process $C_t = V_t - \int_0^t \xi_u dS_u = \xi_0 S_0 + \eta_0 = C_0$ is a constant for all t .
- A complete market is a market where every contingent claim has a hedging portfolio. (ex. Black-Scholes model)
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Q: Then what is a 'good' hedging strategy in an incomplete market?

Model

- Information process

$$dX_t = \alpha(X_t)dt + \beta(X_t)dW_t^X \text{ with } X_0 = 0, \quad (5)$$

where W^X is a standard Brownian motion.

- Price process

$$dS_t = \mu_0 S_{t-} dt + \sigma S_{t-} dW_t + S_{t-} dR_t \text{ with } S_0 = s, \quad (6)$$

where W is another standard Brownian motion with $[W, W^X]_t = \rho t$ and $\mu_0, \sigma > 0$.

Model

→ The jump process R_t is defined by $\sum_{0 < s \leq t} U(X_t) 1_{\Delta R_s(X_s) \neq 0}$, where $U(\cdot)$ is a random function which denotes jump sizes.

→ We assume that $E[U(\cdot)|X_t] := \kappa_1(X_t) \in \mathbb{R}$ and $E[U^2(\cdot)|X_t] := \kappa_2^2(X_t) > 0$. The jump process R_t can be written as $R_t = \int_0^t \int_{-\infty}^{\infty} y p^R(X_s, dy, ds)$,

→ We assume that there exists a compensated measure $m_1(X_t, dy, dt)$ such that $E[\int_0^T C_s dR_t] = E[\int_0^T C_s \int_{\mathbb{R}} y m_1(X_s, dy, ds)]$ for all nonnegative \mathcal{F}_t -adapted processes C_t .

→ Let Radon-Nikodym derivatives be

$$f_i(X_t) = \frac{\int_{\mathbb{R}} y^i m_1(X_t, dy, dt)}{dt},$$

$i = 1, 2$ for a convenience. We also assume that Radon-Nikodym derivatives $f_i(X_t)$ are bounded and \mathcal{F}_t -adapted.

Example

We assume that a random measure $p^R(X_s, dy, ds)$ is structured by a tensor product of random measures such as $N(X_s, s) \times J(X_s, y)$. For example, let $N(X_s, ds)$ be a poisson measure $N(X_t, t) = \#\{0 \leq s \leq t; \Delta R_s(X_s) \in \mathbb{R}/\{0\}\}$ and $J(X_s, y)$ be a random measure associated with a jump size. Then, R_t is a compound Poisson process and the jump counting measure $N(X_t, t)$ is associated with a doubly stochastic Poisson random process with an intensity $\lambda(X_t)$. Under this assumption, we can observe that

$$\begin{aligned} E\left[\int_0^t \int_{\mathbb{R}} y m_1(\cdot, dy, ds) | X_t\right] &= \int_0^t \kappa_1(X_s) \lambda(X_s) ds, \\ E\left[\int_0^t \int_{\mathbb{R}} y^2 m_1(\cdot, dy, ds) | X_t\right] &= \int_0^t \kappa_2^2(X_s) \lambda(X_s) ds. \end{aligned} \tag{7}$$

This implies that both $R_t - \int_0^t \kappa_1(X_s) \lambda(X_s) ds$ and $[R, R]_t - \int_0^t \kappa_2^2(X_s) \lambda(X_s) ds$ are \mathbf{P} -martingale.

- This example contains models which are studied in Lee and Song(2007) and Kang and Lee(2014). If $\lambda(X_s)$ is a constant, the above example coincides with the model in Kang and Lee. If $\kappa_i(X_s)$, $i = 1, 2$ are constants, the above example becomes the model in Lee and Song.

Model

Note that $\hat{R}_t := R_t - \int_0^t f_1(X_s)ds$ is a martingale under \mathbf{P} . Observe that we can write

$$\begin{aligned} dS_t &= \mu_0 S_{t-} dt + \sigma S_{t-} dW_t + S_{t-} dR_t \\ &= \mu_0 S_{t-} dt + \sigma S_{t-} dW_t + S_{t-} dR_t - f_1(X_t)dt + f_1(X_t)dt \\ &= (\mu_0 + f_1(X_t))S_{t-} dt + \sigma S_{t-} dW_t + S_{t-} d\hat{R}_t. \end{aligned} \quad (8)$$

Let

$$M_t = \int_0^t S_{s-} (\sigma dW_s + d\hat{R}_s). \quad (9)$$

We assume that

$$\| [M, M]_T^{1/2} \|_{L^2}^2 < \infty, \quad (10)$$

$$\| \int_0^T |\mu_0 + f_1(X_t)| dt \|_{L^2}^2 < \infty. \quad (11)$$

Then, S_t becomes a \mathcal{H}^2 semimartingale with the canonical decomposition $S_t = M_t + A_t$, and M_t is a square-integrable martingale under \mathbf{P} . We also assume that

$$|\Delta R_t| < \frac{\sigma^2 + f_2(X_t)}{\mu_0 + f_1(X_t)} \quad (12)$$

to avoid a signed measure.

Minimal Martingale Measure

- pricing point of view, the second fundamental theorem
- useful to find the Föllmer-Schweizer decomposition

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Definition

A martingale measure \mathbf{Q} which is equivalent to \mathbf{P} is called *minimal* if $\mathbf{Q} = \mathbf{P}$ on \mathcal{F}_0 , and if any square-integrable \mathbf{P} -martingale L that satisfies $\langle L, M \rangle = 0$ remains a martingale under \mathbf{Q} , where M is the martingale part of S in the canonical decomposition under \mathbf{P} .

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Theorem

Let

$$\Theta_t = - \int_0^t \frac{(\mu_0 + f_1(X_s))}{\sigma^2 + f_2(X_s)} (\sigma dW_s + d\hat{R}_s), \quad (13)$$

and assume that $E[e^{2\Theta_t}] < \infty$ for every $t \leq T$. Let

$$Z_t = 1 + \int_0^t Z_{s-} d\Theta_s. \quad (14)$$

Then, Z_t is positive a.s. and a \mathbf{P} -martingale. Moreover, the probability measure \mathbf{Q} defined by $d\mathbf{Q} = Z_T d\mathbf{P}$ is the minimal martingale measure of S .

Minimal Martingale Measure

Idea of the Proof:

- Doob Meyer Decomposition of M_t
- Girsanov-Meyer theorem
- Kunita-Watanabe inequality
- Uniqueness of SDE
- Stochastic Exponential
- condition on a local martingale to be a true martingale

Q dynamics

Under the minimal martingale measure,

$$\widetilde{W}_t := W_t + \int_0^t \frac{\mu_0 + f_1(X_s)}{\sigma^2 + f_2(X_s)} \sigma ds \text{ and} \quad (15)$$

$$\widetilde{W}_t^X := W_t^X + \rho \int_0^t \frac{\mu_0 + f_1(X_s)}{\sigma^2 + f_2(X_s)} \sigma ds \quad (16)$$

are Brownian motions under \mathbb{Q} .

Q dynamics

Under \mathbf{Q} , the compensated measure of $p^R(X_t, dy, dt)$ is given by

$$q^*(X_t, dy, dt) := p^R(X_t, dy, dt) - \left(1 - \frac{\mu_0 + f_1(X_t)}{\sigma^2 + f_2(X_t)}y\right) m_1(X_t, dy, dt). \quad (17)$$

Moreover, S and X satisfies SDEs

$$dS_t = \sigma S_t d\widetilde{W}_t + S_t d\widetilde{R}_t, \quad (18)$$

$$dX_t = \left(\alpha(X_t) - \rho\sigma \frac{\mu_0 + f_1(X_t)}{\sigma^2 + f_2(X_t)}\right)dt + \beta(X_t)d\widetilde{W}_t^X, \quad X_0 = 0 \quad (19)$$

under measure \mathbf{Q} , where $\widetilde{R}_t := \int_0^t \int_{\mathbb{R}} y q^*(X_t, dy, dt)$.

Local Risk Minimization Strategy

Value process

- ξ_t : the amount of the underlying asset
- η_t : the amount of the money market account
- V_t : the value process of a portfolio (ξ, η) defined by $V_t = \xi_t S_t + \eta_t$

Cost process

- C_t : the cost process defined by $C_t = V_t - \int_0^t \xi_t dS_t$

Local risk minimization strategy in an incomplete market (Föllmer and Schweizer)

- local risk minimization strategies ξ_t : The cost process C is a square integrable martingale orthogonal to M , i.e. $\langle C, M \rangle_t = 0$ where M is the martingale part of S under \mathbf{P} .

Local Risk Minimization Strategy

A sufficient condition for the existence

The existence of an optimal strategy is equivalent to a decomposition

$$H = V_0 + \int_0^T \xi_u^H dS_u + L_T^H$$

where L_t^H is a square integrable martingale orthogonal to M_t . For such a decomposition, the associated optimal strategy (ξ_t, η_t) is given by $\xi_t = \xi_t^H$, $\eta_t = V_t - \xi_t S_t$, where $V_t = V_0 + \int_0^t \xi_u^H dS_u + L_t^H$.

Local Risk Minimization Strategy

Computation of the optimal strategy

Suppose that $V_t = E^{\mathbf{Q}}[H(S_T)|\mathcal{G}_t]$ has a decomposition

$$V_t = V_0 + \int_0^t \xi_u^H dS_u + L_t$$

where L_t is a square integrable \mathbf{P} martingale such that $\langle L, M \rangle_t = 0$ under \mathbf{P} . Then ξ_t^H is given by

$$\xi^H = \frac{d\langle V, S \rangle}{d\langle S, S \rangle}. \quad (20)$$

where the conditional quadratic variations are calculated under \mathbf{P} .

- role of the minimal martingale measure(L_t)

The main theorem

Theorem

Define the Radon-Nikodym derivative $j(t, S_{t-})$ as follows:

$$j(t, S_{t-})dt = \int_{\mathbb{R}} \left(v(t, (1+y)S_{t-}, X_t) - v(t, S_{t-}, X_t) \right) y m_1(X_t, dy, dt). \quad (21)$$

Then, ξ^H is represented by

$$\begin{aligned} \xi_t^H &= \frac{v_s(t, S_{t-}, X_t)\sigma^2}{\sigma^2 + f_2(X_t)} + \frac{v_x(t, S_{t-}, X_t)\sigma\rho\beta(X_t)}{(\sigma^2 + f_2(X_t))S_{t-}} + \frac{j(t, S_{t-})}{(\sigma^2 + f_2(X_t))S_{t-}} \\ &= \frac{\sigma^2}{\sigma^2 + f_2(X_t)} v_s(t, S_{t-}, X_t) + \frac{\sigma\rho\beta(X_t)}{(\sigma^2 + f_2(X_t))S_{t-}} v_x(t, S_{t-}, X_t) + \frac{j(t, S_{t-})}{(\sigma^2 + f_2(X_t))S_{t-}}. \end{aligned} \quad (22)$$

Example

We choose the information process X_t as an OU process which is

$$dX_t = -\tilde{\alpha}_0 X_t dt + \tilde{\beta}_0 dW_t^X \text{ with } X_0 = 0, \quad (23)$$

where $\tilde{\alpha}_0 = 1$ and $\tilde{\beta}_0 = 1$. Note that $E[X_t] = 0$ and X_t is a mean-reversion process. We choose the intensity process $\lambda(X_t) = \lambda_0 + \epsilon_2 |X_t|$ where $\lambda_0 = 10$ and $\epsilon_2 = 200$. In this setting, the jump process has the default jump intensity λ_0 . The coefficient ϵ_2 and $|X_t|$ determine the level of jump opportunity at t . $U(X_t)$ follows a uniform distribution with interval $[-\epsilon_1 + \kappa_1(X_t), \epsilon_1 + \kappa_1(X_t)]$. Let $\kappa_1(X_t) = 0.3 \cdot X_t$ and $\epsilon_1 = 0.03$. Then we obtain $\kappa_2^2(X_t) = \frac{\epsilon_1^2}{3} + \kappa_1^2(X_t)$. The average of jump sizes is $\kappa_1(X_t)$. The coefficient ϵ_1 can be interpreted as the error of the information effect. Since our model must satisfy the upper boundedness of the jump size (We discuss it in (12) later), we choose $\sigma = 0.1$ and $\epsilon_1 = 0.03$.

$E[(C_T - C_0)^2]$	
Black Scholes Model	Two Factor Model
0.179358	0.054593

Table: Hedging Error : $S_0 = 100$, $\sigma_0 = 0.2$, $K = 100$, $T = 1$, $dt = \frac{1}{100}$

Example

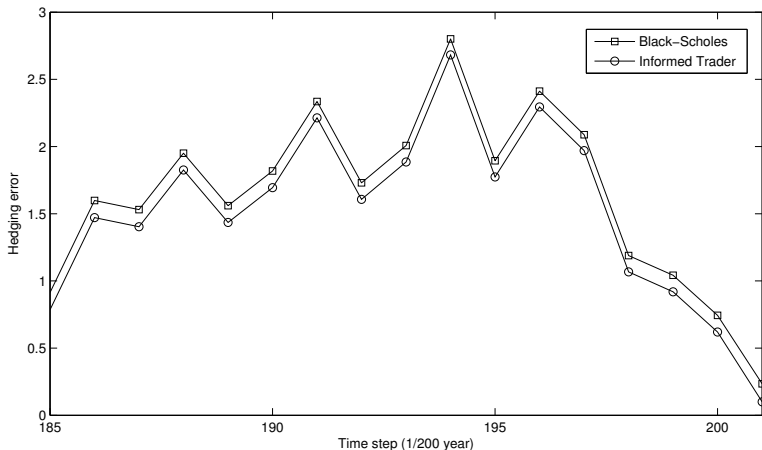


Figure: Sample of Hedging strategies, $\int_0^{k/200} \eta_s^{n,H} ds - V_T + C_0$ ($S_0 = 100$, $\sigma_0 = 0.2$, $K = 100$, $T = 1$, $dt = \frac{1}{200}$, $k = 190, \dots, 200$)

What to do next?

What to do next?

- more microstructure \rightarrow algorithmic trading/ HFT
- other problems on information asymmetry
- uninformed or less informed trader's learning dynamic
- real data fitting??