Contingent convertible bonds with the default risk premium

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Objectives

1. Pricing Contingent Convertible bonds (CoCos)
   - Modelling a ratio of the common-equity Tier1 (CET1) capital under Basel 3 framework
   - Deriving the pricing formula of CET1 ratio trigger CoCos as a semi-analytic form
2. Quantifying post-conversion risk of CoCos
   - Investigating additional risk that should be considered when adopting an equity-conversion type for a loss absorbing method
   - Quantifying post-conversion risk premium
3. Finding post-conversion risk premium in the real market

Contingent Convertible Bonds (CoCos)

- *Contingent convertible bonds*, CoCos for short, is a new-styled bond that is automatically converted into equities or is written down when the capital ratio falls below a specified level.
- When conversion (either equity-conversion or write-down) is activated, a bank can be mechanically recapitalized by CoCos without the injection of new external funds.
- Due to the risk caused by conversion, it provides higher coupons than the straight bond.
  - Convertible bond: a holder can exercise conversion
  - CoCo bond : mandatory conversion

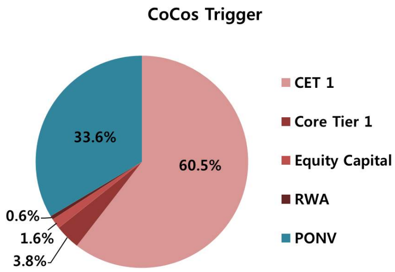
Main design features of CoCos

Trigger that activates loss absorption process

- Capital-ratio trigger
- Regulatory trigger

Loss absorption mechanism at the moment of trigger

- Equity-conversion (EC)
- Write-down (WD)



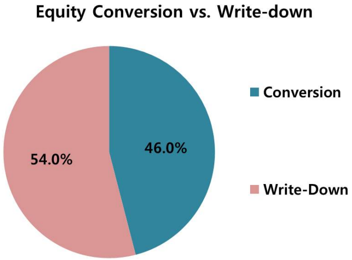


Figure: By measure of trigger and loss absorption mechanism (Source: Moody's)

Objective 1

- Model for pricing CET1 ratio trigger CoCos

Payoff of CoCo

Consider a zero-coupon CET1 ratio trigger CoCo with the maturity T, the face value N, the conversion price C_p.

Assume that the equity-conversion fraction wN is converted into C_r shares, and $(1-w)N$ is written-down to $\delta(1-w)N$ with the recovery rate δ.

- The conversion ratio C_r is determined as $C_r = wN/C_p$.

Either one of two possible scenarios is realized depending on the CET1 ratio level of the issuing bank.

- If a CET1 ratio falls down to the trigger level before maturity T, the investor receives

  $$C_r \times S_\tau + \delta(1-w)N,$$

  at trigger time τ.
- Otherwise, the investor receives the entire face value N at maturity T.

Model for CET1 ratio

- The CET1 ratio is estimated as

$$\text{CET1 ratio} = \frac{\text{CET1 capital}}{\text{Total RWAs}} \approx \frac{S_t \times M}{\text{Total RWAs}} = \frac{S_t}{\text{Total RWAs/M}}$$

  where M is a total number of shares issued by a bank.
  - The equity value is observable daily.
  - However, the total RWA value is announced at most quarterly.
- We model the equity value S_t as geometric Brownian motion, and the Total RWAs/M as a single random variable.

Model for CET1 ratio

- S_t : Equity price process with geometric Brownian motion(GBM)

$$dS_t = rS_t dt + \sigma S_t dW_t$$

- L : RWA-per-share value which is a nonnegative random variable with a distribution F and is independent of W_t
- τ_B : Trigger time which is defined as the first passage time when the CET1 ratio S_t/L falls below the level α_0

$$\tau_B = \inf\left\{t \geq 0 : \frac{S_t}{L} \leq \alpha_0\right\} = \inf\left\{t \geq 0 : S_t \leq \alpha_0 L\right\}$$

- The true value of RWA-per-share reveals only at time of trigger, but it can be progressively estimated by using information at the valuation date.

Description of Idea

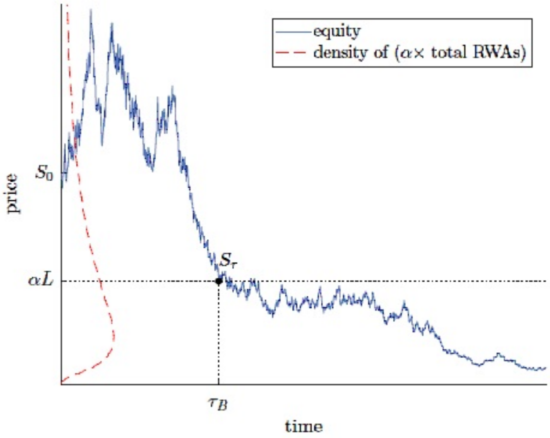


Figure: Model description

Valuation of CoCos: Formula for Price

Define a zero coupon CoCo payoff at conversion $\bar{K}(S_{\tau_B}) = C_r \times S_{\tau_B} + (1-w)\delta$

Theorem 1. (Zero-coupon CoCo)

Suppose that a zero coupon CoCo bond with an unit face value and a maturity T. Let $\bar{K}(\cdot)$ be the contractual payoff of a CoCo bond with $N=1$, and $D(t,s)$ be a discount factor with a constant risk-free rate r. Then, the value of the zero coupon CoCo bond as seen from time $0 \le t \le T$, is given by

$$\bar{P}^{ZC}(t,T) = \int_{\frac{m_t}{\alpha_0}}^{\frac{S_0}{\alpha_0}} \bar{K}(\alpha_0 x) D(t, \tau_B^x) dF(x) + \frac{\bar{K}(S_0)}{D(0,t)} \left\{ 1 - F\left(\frac{S_0}{\alpha_0} \right) \right\}$$
$$+ \int_0^{\frac{m_t}{\alpha_0}} \bar{K}(\alpha_0 x) \int_t^T D(t,s) h(s-t; \alpha_0 x, S_t) ds dF(x) + D(t,T)\{1 - G_t(T)\}$$

where $G_t(T)$ is the conditional probability distribution of trigger time τ_B and m_t is running minimum of equity price over time $[0,t]$, $m_t = \min_{0\le v\le t} S_v$. The function h is the probability density function of trigger time.

Probability of trigger time

Fact (First hitting time)

Let S_t satisfy GBM. Define a stopping time τ as the first time at which S_t reaches a barrier $B < S_0$. Then the distribution H of τ, $\mathbb{E}[1(\tau \leq t)]$, is given by

$$H(t; B, S_0) = \Phi\left(\frac{\ln(\frac{B}{S_0}) - \left(r - \frac{1}{2}\sigma^2\right) t}{\sigma\sqrt{t}}\right) + \left(\frac{B}{S_0}\right)^{\frac{2r}{\sigma^2}-1} \Phi\left(\frac{\ln(\frac{B}{S_0}) + \left(r - \frac{1}{2}\sigma^2\right) t}{\sigma\sqrt{t}}\right)$$

where Φ is a standard normal.

Lemma (Probability of trigger time)

For $s \geq t$, the conditional distributions $G_t(s)$ of trigger time τ_B is given by

$$G_t(s) = \mathbb{Q}(\tau_B \leq s|\mathcal{F}_t) = 1 - F\left(\frac{m_t}{\alpha_0}\right) + \int_0^{\frac{m_t}{\alpha_0}} H(s - t; \alpha_0 x, S_t) dF(x),$$

where F is the cumulative distribution of RWA level L, $m_t = \min_{0 \leq v \leq t} S_v$.

Objective 2

- Model for measuring post-conversion risk of CoCos

Different Market Reaction on issuing EC/WD CoCos

- Avdjiev et. al. [1] shows how CoCo issuance affects to reduce insolvency risk of an issuing bank.
- CDS spread of issuing bank at the issue announcement dates depending on the following type of CoCos
  - loss absorption mechanism : EC vs. WD
  - trigger level : greater than 6% vs. less than 6%
- The issuance of EC CoCo gives a significantly positive effect to CDS spread of CoCo issuer, however, that of WD CoCo does not.
- For EC CoCo, the issuance of high-trigger CoCo affects stronger positive effect than that of low-trigger CoCo to CDS spread

Different Risk Profile of EC/WD CoCos

- The empirical study by Avdjiev et. al. [1] implies that in terms of reducing credit risk,
- (Loss absorption mechanism : EC vs. WD)
  - For issuers, EC CoCo is more efficient than WD CoCo
  - For investors, EC CoCo is riskier than WD CoCo
- (Trigger level : high-trigger vs. low-trigger)
  - For issuers, EC CoCo with high-trigger is more efficient than that with low-trigger
  - For investors, EC CoCo with high-trigger is riskier than that with low-trigger
- In most of the previous studies, the issue on market preferences arising from the different design of CoCos has not been dealt with in pricing CoCos.

Post-conversion risk

We define the post-conversion risk as additional risk due to a non-zero default probability of an issuing bank in the post-conversion period.

- In the ex-ante conversion, the risk factors to which EC- and WD CoCos are exposed are same if all else conditions are equivalent.
- In the ex-post conversion, the different processes are activated.

WD CoCo investor

- The bond contract terminates

EC CoCo investor

- Turning into a shareholder with # of equities of the issuing bank

Post-conversion risk of EC

- The converted equities are retained or be able to be liquidated in the market
- EC investor should take additional risk

Model for Adjusted CET1 ratio

- At the moment of trigger τ_B with equity-conversion, C_r shares of stock are converted from CoCo notional wN.
- The adjusted CET1 ratio is modeled as

$$\text{Adjusted CET1 ratio} \approx \frac{S_t \times (M + C_r)}{\text{Total RWAs}} = \frac{S_t}{\text{Total RWAs}/(M + C_r)}$$

- Define L' as the value of the adjusted RWA-per-share in the post-conversion

$$L' = \frac{\text{Total RWAs}}{M + C_r}$$

Regulatory default in ex-post conversion

- Define regulatory default time τ_D
- The first passage time when the adjusted CET1 ratio falls below a certain barrier level α_1 after the conversion τ_B happens,

$$\tau_D = \inf\left\{t \geq \tau_B : \frac{S_t}{L'} \leq \alpha_1\right\} = \inf\left\{t \geq \tau_B : S_t \leq \alpha_1 \psi(L)\right\},$$

where

- α_1 is set as the level less than $\alpha_0 L/\psi(L)$, so that $\alpha_1\psi(L) < \alpha_0 L$
- ψ is a monotonic function defined as

$$\psi(x) = \frac{x}{1 + C_r(x)/M}.$$

Regulatory default in ex-post conversion

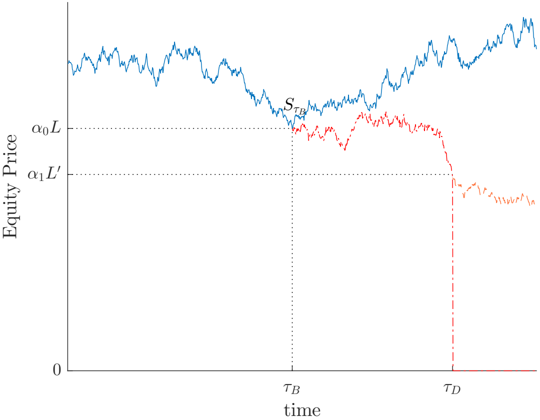


Figure: Simulation of sample paths of an equity price under the proposed model

Quantification of Post-conversion risk

- To reflect the possibility of regulatory default in ex-post conversion to the CoCo price, we build up the hypothetical payoff on $[\tau_B, T]$
- The idea of the down-and-out option pricing is used
- Let ζ^{DO} be a down-and-out call option price with strike $E = 0$ and a barrier $B = \alpha_1 \psi(L)$.

Hypothetical payoff $K(\tau_B, S_{\tau_B})$

$$K(\tau_B, S_{\tau_B}) = C_r(S_{\tau_B})\zeta^{DO}(T - \tau_B, \alpha_1\psi(L), S_{\tau_B}) + (1 - w)\delta N$$

$$\zeta^{DO}(T-t, B, S_t) = S_t \left[\Phi\left(\frac{\log\left(\frac{S_t}{B}\right) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \right) - \left(\frac{B}{S_t}\right)^{1+\frac{2r}{\sigma^2}} \Phi\left(\frac{-\log\left(\frac{S_t}{B}\right) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \right) \right]$$

CoCo price with post-conversion risk

Theorem 2. (Zero coupon CoCo with post-conversion risk)

Suppose that a zero coupon CoCo bond with an unit face value and maturity T. The hypothetical CoCo price which reflects post-conversion risk on $[\tau_B, T]$ is given as

$$
\begin{aligned}
& P^{ZC}(t,T) \\
=& (1-w)\delta \int_{\psi^{-1}(m_t/\alpha_1)}^{\infty} D(t,\tau_B^x)dF(x) + \frac{(1-w)\delta}{D(0,t)}\left(F\left(\psi^{-1}\left(\frac{m_t}{\alpha_1}\right)\right) - F\left(\frac{S_0}{\alpha_0}\right)\right) \\
&+ C_r(S_0)\int_{S_0/\alpha_0}^{\psi^{-1}(m_t/\alpha_1)} \zeta^{DO}(S_t, \alpha_1\psi(x), T-t)dF(x) + \int_{m_t/\alpha_0}^{S_0/\alpha_0} K(\tau_B^x, \alpha_0 x)D(t,\tau_B^x)dF(x) \\
&+ \int_0^{\frac{m_t}{\alpha_0}}\int_t^T K(s,\alpha_0 x)D(t,s)h(s-t;\alpha_0 x, S_t)ds dF(x) + D(t,T)\left\{1 - G_t(T)\right\}.
\end{aligned}
$$

where K is the hypothetical payoff.

Post-conversion risk premium

- Define risk premium of CoCo for taking post-conversion risk

$$\text{Risk premium } \mathcal{P}(\alpha_1) = -\frac{1}{T} \ln \frac{P^{ZC}(0, T)}{\bar{P}^{ZC}(0, T)}$$

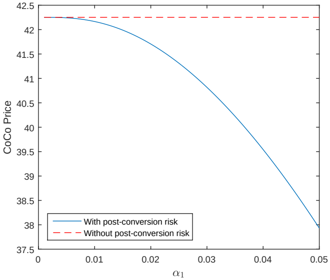
  where $\bar{P}^{ZC}$ and P^{ZC} are T-maturity zero coupon CoCo prices without and with post-conversion risk, respectively.
- The risk premium $\mathcal{P}(\alpha_1)$ represents the extra yield of CoCos with post-conversion risk over the yield of CoCo without post-conversion risk given level α_1.

Objective 3

- Numerical test and finding post-conversion risk premiums

Post-conversion risk premium

- We compare EC CoCo prices and yield-to-maturity(YTM) with and without post-conversion risk depending on the level α_1



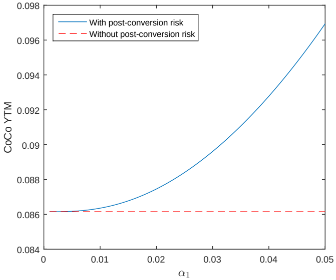
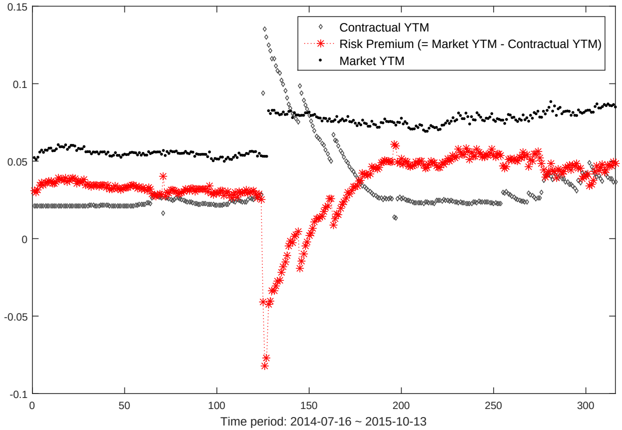


Figure: Comparison of CoCo bond prices (left) and CoCo YTMs (right) for floored conversion prices with post-conversion risk against post-conversion risk

Post-conversion risk premium

Extracting post-conversion risk premium from market YTM of CoCo issued by Credit Suisse(ISIN: XS0810846617) based on the corresponding theoretical YTM with contractual payoff



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- De Spiegeleer, J., Schoutens, W., Pricing contingent convertibles: a derivatives approach, *The Journal of Derivatives*, 20: 27-36, 2012.
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