Optimal Contracting and Consumption/Portfolio Selection with Limited Commitment

Hyeng Keun Koo

Financial Engineering, Ajou University

April 24, 2017

The 5th Asian Quantitative Finance Conference, Seoul Korea

Contents

Contents:

Dynamic Contracting with Limited Commitment

An optimal insurance contracting problem with limited commitment

- Two agents:
  (i) a risk-neutral principal,
  (ii) a risk-averse agent who receives a stream of stochastic income $(I_t)_{t=0}^{\infty}$ with a utility function $V_0 = \mathbb{E}\left[\int_0^{\infty} e^{-\delta t} u(C_t) dt\right]$.
- : Symmetric information
- The principal's problem: Choose $(C_t)_{t=0}^{\infty}$ to maximize

$$\mathbb{E}\left[\int_0^{\infty} e^{-rt} (I_t - C_t)\ dt\right]$$

subject to
(i) $V_0 \geq \bar{V}$ (an outcome of the bargaining game between the principal and the agent)
(ii) (Limited Commitment)

$$V_t = \mathbb{E}\left[\int_t^{\infty} e^{-\delta(s-t)} u(C_s)\, ds | \mathcal{F}_t\right] \geq \mathbb{E}\left[\int_t^{\infty} e^{-\delta(s-t)} u(I_s)\, ds | \mathcal{F}_t\right] \quad \forall t \in [0, \infty).$$

Dynamic Contracting with Limited Commitment

Answer to the classical problem with full commitment:
$C_t = \left(u^{'}\right)^{-1}\left(\lambda e^{(\delta - r)t}\right)$ such that

$$V_0 = \int_0^\infty e^{-\delta t} u(C_t)dt = \bar{V}.$$

- If, in particular, $\delta = r$, then C_t is constant.
- The agent's consumption stream doesn't have any risk$\rightarrow$ the principal bears all the risk.

Literature

Early Research on International Lending and Wage Contracts

- Eaton and Gersovitz (1981, RES), Debt with Potential Repudiation: Theoretical and Empirical Analysis: debt based on reputation without collateral.
- Bulow and Rogoff (1989, AER), Sovereign Debt: Is To Forgive To Forget?: a simple exclusion from the credit market does not allow non-zero borrowing.
- Thomas and Worrall (1988, RES), Self-Enforcing Wage Contracts: backloading of wages

Literature on Limited Commitment

- Limited commitment to repay debt, autarky
- Asset Markets and Pricing: Kehoe and Levine (1993), Alvarez and Jermann (2000, 2001), Azariadis and Kass (2007), Ábraham and Cárceles-Poveda (2008), Hellwig and Lorenzoni (2009), Osambela (2012), Azariadis and Choi (2012, 2013), Bidian and Bejan (2012), Werner (2012)
- International Finance Kehoe and Perri (2002, 2004)
- Political Economy: Acemoglu, Golosov and Tsyvinski (2008)

Literature

Continuous-time Optimal Contracting with Limited Commitment

- Grochulski and Zhang (2011, JET), Optimal Risk Sharing and Borrowing Constraints in a Continuous-Time model with Limited Commitment.
- Miao and Zhang (2015, JET), A Duality Approach to Continous-Time Contracting Problem with Limited Commitment.

Applications to Corporate Finance and Risk Managemnt

- Ai and Li (2015, JFE), Investment and CEO Compensation under Limited Commitment.
- Bolton, Wang, and Yang (2016, WP), Liquidity and Risk Management: Coordinating Investment and Compensation Policies

Grochulski and Zhang's Solution

Review of Grochulski and Zhang(2011)'s Solution

Assumptions:

(i) $r = \delta$

(ii) I_t follows the following geometric Brownian motion

$$\frac{dI_t}{I_t} = \mu_I dt + \sigma_I dB(t),$$

where $B(t)$ is a standard Brownian motion.

Consider the case where

$$u(c) = \begin{cases} \frac{c^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ \log c & \text{if } \gamma = 1. \end{cases}$$

Grochulski and Zhang's Solution

Review of Grochulski and Zhang(2011)'s Solution x

Let

$$m_t = \max\left(m_0, \max_{0\leq u\leq t} I_u\right),$$

where m_0 is determined by $\bar{V}$.

Then, for a certin constant $C > 0$

$$C_t = Cm_t.$$

$\rightarrow$ Only partial risk sharing.

Grochulski and Zhang's Solution

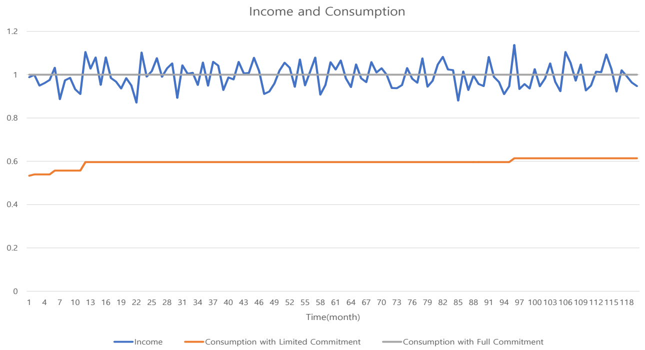


Figure: *Optimal consumption process* $(r = \delta = 0.03, \mu_I = 0, \sigma_I = 0.2.\gamma = 2.)$

Grochulski and Zhang's Solution

Implementation of the Solution

- Principal
  - (i) sets the initial wealth X_0 of the agent,
  - (ii) provides futures contracts to the agent,
  - (iii) sets the credit(borrowing) limits L_t at each point of time such that the agent's wealth $X_t \geq -L_t$.
- Agent
  - (i) chooses his/her consumption
  - (ii) is allowed to save at the interest rate equal to r.
  - (iii) the amount of hedging by using the futures

  subject to the wealth constraint $X_t \geq -L_t$ for each $t \in [0, \infty)$.

Thus, the *optimal insurance problem with limited commitment* is equivalent to the *consumption and hedging problem with endogenous credit limits*!

Optimal Consumption/Portfolio Selection with Limited Commitment

Our work (joint work with Kyung Jin Choi, Byunghwa Lim, and Jane Yoo) is the consumption/portfolio selection problem related to the contracting problem:

- we explore the characteristic of the optimal consumption profile, relating it to the full comitment case,
- we explore the hedging/portfolio selection in detail,
- we introduce an asset with aggregate risk (relevant to international lending and corporate finance, cf. Bolton, Wang and Yang 2016).

Optimal Consumption/Portfolio Selection with Limited Commitment

The agent's problem in the previous page when there exists a risky asset with a positive risk premium. The price S_t of the risky asset evolves according to

$$\frac{dS_t}{S_t} = \mu dt + \sigma dB(t).$$

We consider the following participation constraints of more general type:

$$\mathbb{E}_t \left[\int_t^\infty e^{-\delta s} u(c_s) ds \right] \geq \phi^{1-\gamma} \cdot \mathbb{E}_t \left[\int_t^\infty e^{-\delta s} u(I_s) ds \right], \qquad \text{for every } t \geq 0,$$

with $0 \leq \phi \leq \bar{\phi}$.

Optimal Consumption

We say (e_t) *super-replicates* $(\overline{e}_t)$ if $e_t \geq \overline{e}_t$ for every $t \geq 0$, and that (e_t) is a *minimum super-replicating strategy* for $(\overline{e}_t)$ subject to a set of constraints, if (i) (e_t) satisfies the constraints in the set, (ii) it super-replicates $(\overline{e}_t)$, and (iii) $\mathbb{E}\left[\int_0^\infty H_t e_t dt\right]$ is the smallest among all the strategies satisfying (i) and (ii) (Here, $H_t \equiv \exp\left(-\left\{r + \frac{1}{2}\theta^2\right\}t - \theta B_t\right)$, with $\theta \equiv \frac{\mu - r}{\sigma}$.).
Similar to El-Karoui and Jeanblanc-Piqué (1998) we can show the following proposition is true.

Proposition

Optimal consumption c_t^* *is a minimum super-replicating strategy for the initial consumption policy subject to participation constraints.*

Optimal Wealth Process

Optimal wealth process is given by

$$X_t^* = \frac{I_t}{K}\left(\frac{y_t^*}{1+D_t^*}\right)^{-\frac{1}{\gamma}} - \frac{I_t}{r_I} + \frac{\phi^{1-\gamma}\alpha_+ Q^{\gamma\alpha_+}}{\hat{\delta}(\alpha_+-1)(1-\gamma+\alpha_+\gamma)}\left(\frac{y_t^*}{1+D_t^*}\right)^{\alpha_+-1} I_t,$$

and satisfies

$$X_t^* \geq -m^* I_t, \qquad \text{for every } t \geq 0,$$

where

$$K \equiv r + \frac{\delta - r}{\gamma} + \frac{(\gamma-1)}{2\gamma^2}\theta^2 > 0,$$

$$m^* \equiv \frac{1}{r_I} - \frac{(1-\gamma+\alpha_+\gamma)}{\gamma K(\alpha_+-1)} Q \geq 0, Q \equiv \delta\left(\frac{K\alpha_+\gamma}{(1-\gamma+\gamma\alpha_+)\hat{\delta}}\right)^{\frac{1}{1-\gamma}} < 1,$$

$$D_t^* = \max\left(0, \max_{0\leq s\leq t} Q^\gamma \cdot y_s^* - 1\right), y_t \equiv \lambda^* e^{\delta t} H_t I_t^\gamma,$$

with α^+ and λ^* being solutions to appropriate algebraic equations.

Optimal Portfolio

Optimal portfolio is given by

$$\pi_t^* = \frac{\theta}{\sigma\gamma}\left(X_{2,t}^* + \frac{I_t}{r_I}\right) - \frac{\sigma_I}{\sigma r_I} I_t + \frac{(\gamma\sigma_I - \theta)\phi^{1-\gamma}\alpha_+ Q^{\gamma\alpha_+}}{\gamma\sigma(\alpha_+ - 1)\hat{\delta}}\left(\frac{y_t^*}{1+D_t^*}\right)^{\alpha_+ - 1} I_t.$$

The terms on the Right side of the equation

1. The first term: the myopic demand,
2. The second term: hedge against has changes in income risk
3. The third term:

$$\frac{\sigma_I \phi^{1-\gamma}\alpha_+ Q^{\gamma\alpha+}}{\sigma(\alpha_+ - 1)\hat{\delta}}\left(\frac{y_t^*}{1+D_t^*}\right)^{\alpha_+ - 1} \cdot I_t - \frac{\theta Q^{\gamma\alpha+}}{\gamma\sigma(\alpha_+ - 1)\hat{\delta}}\left(\frac{y_t^*}{1+D_t^*}\right)^{\alpha_+ - 1} \cdot I_t.$$

Relationship with Exogenous Borrowing Constraints

We consider the following problem considered previously by He and Pagés (1993, ET), El-Karoui and Jeanblanc-Piqué (1998, FS), Ahn, Choi, and Lim (2016, forthcoming in JFQA):

Problem (Exogenous Credit Limits)

The agent wants to maximize expected utility by selecting the consumption rate and portfolio of assets at each point of time, subject to the budget constraint with initial wealth X_0 and the following credit limits

$$X_t \geq -\mathcal{D} I_t, \qquad t \geq 0, \tag{1}$$

where $0 \leq \mathcal{D} < 1/r_I$ ($r_I \equiv r - \mu_I + \theta\sigma_I$).

Relationship with Exogenous Borrowing Constraints

Proposition

For a given ϕ, $0 < \phi < \bar{\phi}$, the ratio, m^*, of the endogenous credit limit to income has the following properties

(a) *m^* is linear and decreasing in δ.*

(b) *(cf. Bulow and Rogoff (1988)) Suppose that $\phi = \bar{\phi}$, where*

$$\bar{\phi} \equiv \frac{1}{r_I} \frac{\gamma K(\alpha_+ - 1)}{(1-\gamma+\alpha_+\gamma)} \left(\frac{K\alpha_+\gamma}{(1-\gamma+\alpha_+\gamma)\hat{\delta}} \right)^{-\frac{1}{1-\gamma}}. \qquad (2)$$

Then,

$$m^* = 0.$$

(c) *For Problem [exogenous borrowing], there exists a unique $\phi_{\mathcal{D}}$ such that it has the same solution, i.e., optimal consumption and portfolio, as the original problem with $\phi = \phi_{\mathcal{D}}$, where*

$$\phi_{\mathcal{D}} \equiv \frac{\gamma K(\alpha_+ - 1)(1/r_I - \mathcal{D})}{(1-\gamma+\alpha_+\gamma)} \left(\frac{K\alpha_+\gamma}{(1-\gamma+\alpha_+\gamma)\hat{\delta}} \right)^{-\frac{1}{1-\gamma}}.$$

Comparative Statics

Proposition

(cf. Eaton and Gersovitz (1981)) The ratio, m^*, of the endogenous credit limit to income has the following properties.

(a) *m^* is increasing in the mean return of income growth, μ_I, if and only if*

$$F_1(\mu_I) \equiv \frac{1}{\hat{\delta} - r_I + (\alpha_+ - 1/2)(\gamma\sigma_I - \theta)^2} + \alpha_+(\alpha_+ - 1)\left(\frac{1}{\hat{\delta}} - \frac{1}{r_I^2 R}\right) < 0.$$

(b) *m^* is decreasing in the volatility of income growth, σ_I, if and only if*

$$F_2(\sigma_I) \equiv \frac{\gamma(\alpha_+ - 1)(\alpha_+\gamma\sigma_I - \alpha_+\theta + \gamma\sigma_I - \sigma_I) - \theta\alpha_+}{\hat{\delta} - r_I + (\alpha_+ - 1/2)(\gamma\sigma_I - \theta)^2} - \alpha_+(\alpha_+ - 1)(1 - \gamma + \gamma\alpha_+)\left(\frac{\gamma\sigma_I}{\hat{\delta}} + \frac{\theta}{r_I^2 R}\right) < 0.$$

$$R \equiv \frac{(1 - \gamma + \alpha_+\gamma)\delta}{\gamma K(\alpha_+ - 1)} Q.$$

Comparative Statics

Proposition

(Continued from the previous page)

(c) m^* *is decreasing in the Sharpe ratio,* θ*, if and only if*

$$F_3(\theta) \equiv \frac{\sigma_I + (\alpha_+ - 1)(\gamma\sigma_I - \theta)}{\hat{\delta} - r_I + (\alpha_+ - 1/2)(\gamma\sigma_I - \theta)^2} + (\alpha_+ - 1)(1 - \gamma + \gamma\alpha_+)\left(\frac{\theta}{\gamma K} - \frac{\sigma_I}{r_I^2 R}\right) < 0.$$

(d) m^* *is decreasing in the risk-free rate,* r*, if and only if*

$$F_4(r) \equiv \frac{\sigma - \sigma_I + (\gamma\sigma_I - \theta)(1 - \alpha_+)}{\hat{\delta} - r_I + (\alpha_+ - 1/2)(\gamma\sigma_I - \theta)^2} + (\alpha_+ - 1)(1 - \gamma + \gamma\alpha_+)\left(\frac{\gamma\sigma - \theta}{\gamma K} - \frac{\sigma - \sigma_I}{r_I^2 R}\right) < 0.$$

Stochastic Investment Opportunity

We consider a regime switching model.

- There exist two regimes, regime H and regime L: the market parameters are given by $(r_i, \mu_i, \sigma_i), \quad i \in \{H, L\}$.The regimes change according to a Markov switching process with constant intensities.
- There exist contracts which can be used to hedge against regime changes. The contracts are priced with an equivalent martingale measure $\tilde{\mathbb{Q}}$ under which each Poisson process has an intensity equal to a constant, $\tilde{\zeta}_i$, for $i \in \{H, L\}$.

Stochastic Investment Opportunity

Theorem

Under certain assumptions the endogenous credit limit is determined by

$$\bar{L}_t^i = m_i^* I_t$$

for each regime $i \in \{H, L\}$, *where* m_H^* *and* m_L^* *are defined appropriately:*

Methodology: A Dual Approach

Our method consists of the following steps:

- A dual martiganle approach by writing the Lagrangian of the opmization problem introducing appropriate Lagrange multipliers (cf. He and Pagés 1993, El Karoui and Jeanblanc-Piqué 1998, Miao and Zhang 2016),
- Estabilish the duality between the dual value function and the value function,
- Transform the minimization problem in the duality into a series of optimal stopping problems (similar to Dybvig and Rogers 2013, Dybvig, Jang, Koo 2015).

Methodology: A Dual Approach

Consider the following Lagrangian:

$$\mathcal{L} \equiv \mathbb{E}\left[\int_0^\infty e^{-\delta t} u(c_t)dt\right] + \lambda\left(x - \mathbb{E}\left[\int_0^\infty H_t(c_t - I_t)dt\right]\right) + \mathbb{E}\left[\int_0^\infty \mathbb{E}_t\left(\int_t^\infty e^{-\delta s}\left\{u(c_s) - \delta^{1-\gamma}u(I_s)\right\}ds\right)dD_t\right], \quad (3)$$

where D_t is a non-decreasing real-valued process with $D_0 = 0$ and dD_t being the (infinitesimal) Lagrange multiplier for the participation constraint at time t. Notice that if D_t has a derivative $\dot{D}_t$, then the last term of the right hand side in (3) can be written as

$$\mathbb{E}\left[\int_0^\infty \dot{D}_t \mathbb{E}_t\left(\int_t^\infty e^{-\delta s}\left\{u(c_s) - \delta^{1-\gamma}u(I_s)\right\}ds\right)dt\right],$$

where $\dot{D}_t$ is the ordinary Lagrange multiplier for the constraint at t. D_t, however, is not generally differentiable. By changing the order of integration it can be written as

$$\mathbb{E}\left[\int_0^\infty e^{-\delta t}\left(\int_0^t dD_s\right)\left\{u(c_t) - \delta^{1-\gamma}u(I_t)\right\}dt\right] = \mathbb{E}\left[\int_0^\infty e^{-\delta t} D_t\left\{u(c_t) - \delta^{1-\gamma}u(I_t)\right\}dt.\right].$$

Methodology: A Dual Approach

The Lagrangian can be written as

$$\mathcal{L} \equiv \mathbb{E}\left[\int_0^\infty e^{-\delta t}u(c_t)dt\right] + \lambda\left(x - \mathbb{E}\left[\int_0^\infty H_t(c_t - I_t)dt\right]\right) + \mathbb{E}\left[\int_0^\infty e^{-\delta t}\left\{u(c_t) - \delta^{1-\gamma}u(I_t\right\} D_t dt\right].$$

$$\widetilde{V}(x;\lambda,D) = \max_{c_t}\left\{\mathbb{E}\left[\int_0^\infty e^{-\delta t}u(c_t)dt\right] - \lambda\mathbb{E}\left[\int_0^\infty H_t c_t dt\right] + \mathbb{E}\left[\int_0^\infty e^{-\delta t}D_t\left(u(c_t) - \delta^{1-\gamma}u(I_t)\right)dt\right]\right\} \tag{4}$$

Methodology: A Dual Approach

Denoting the inverse of marginal utility $u^{'}(\cdot)$ by $L(\cdot)$, the first order condition for the optimization problem in (4) is given as

$$c_t^* = L\left(\frac{\lambda e^{\delta t} H_t}{1 + D_t}\right).$$

Shadow price D_t

(Intuition) The role of the shadow price D_t in keeping the participation constraint satisfied:

1. Suppose that the participation constraint binds and there is a negative shock from the market, i.e., $dH_t > 0$ or a positive shock to the income process, i.e., $dI_t > 0$.
2. In the absence of adjustment in the consumption policy, the participation constraint would be violated.
3. The shadow price is adjusted upward $(dD_t > 0)$ to prevent this from happening.

Optimal Stopping Problems

We transform the problem into a series of optimal stopping problems

- Similar to El Karoui and Jeanblanc-Piqué (1998), Dybvig and Rogers (2013),
- Use the equivalence of the following
  - For every $t > 0$, the choice of an increasing right-continous process D_t
  - For every $v > 0$, the choice of the first time, τ_v, for D_t to reach v.

Optimal Stopping Problems

Define a function

$$f(1+D_t) := e^{-\delta t}\left\{(1+D_t)\tilde{u}\left(\frac{\lambda e^{\delta t}H_t}{1+D_t}\right) - (1+D_t)\delta^{1-\gamma}u(I_t)\right\},$$

then the dual value function is rewritten as

$$\begin{aligned}
\Psi(\lambda) = &\inf_{\{D_t\}} \mathbb{E}\left[\int_0^\infty \left(\int_0^\infty \mathbf{1}_{\{D_t \geq v\}} f'(1+v)dv + \underbrace{e^{-\delta t}\left(\tilde{u}(\lambda e^{\delta t}H_t) - \delta^{1-\gamma}u(I_t)\right)}_{:=f(1)}\right) dt\right] \\
&+ \lambda \mathbb{E}\left[\int_0^\infty H_t I_t dt\right] + \mathbb{E}\left[\int_0^\infty e^{-\delta t}\delta^{1-\gamma}u(I_t)dt\right] + \lambda x \\
= &\inf_{\{D_t\}} \mathbb{E}\left[\int_0^\infty \left(\int_0^\infty \mathbf{1}_{\{D_t \geq v\}} f'(1+v)dv\right) dt\right] + J(\lambda) + \lambda x \\
= &\inf_{\{\tau_v\}} \mathbb{E}\left[\int_0^\infty \left(\int_0^\infty \mathbf{1}_{\{\tau_v \leq t\}} f'(1+v)dv\right) dt\right] + J(\lambda) + \lambda x
\end{aligned}$$

where the equalities hold, since

$$f(1+D_t) - f(1) = \int_0^{D_t} f'(1+v)dv = \int_0^\infty \mathbf{1}_{\{D_t \geq v\}} f'(1+v)dv = \int_0^\infty \mathbf{1}_{\{\tau_v \leq t\}} f'(1+v)dv.$$

Variational Inequality

Alternatively, we can solve for the dual value function by solving the variational inequality as in Miao and Zhang (2015): Find $J(z,I) \in \mathcal{C}^2$ such that $J_z(z,I) \geq U_0(I)$ and

$$\widetilde{u}(z)+zI+\frac{1}{2}\sigma_I^2\frac{\partial^2 J}{\partial I^2}+\theta\sigma_I\frac{\partial^2 J}{\partial I \partial z}+\frac{1}{2}\theta^2\frac{\partial^2 J}{\partial z^2}+\mu_I\frac{\partial J}{\partial I}+(\delta-r)\frac{\partial J}{\partial z}-\delta J = 0 \text{ if } \frac{\partial J}{\partial z} > U_o(I),$$

and

$$\widetilde{u}(z)+zI+\frac{1}{2}\sigma_I^2\frac{\partial^2 J}{\partial I^2}+\theta\sigma_I\frac{\partial^2 J}{\partial I \partial z}+\frac{1}{2}\theta^2\frac{\partial^2 J}{\partial z^2}+\mu_I\frac{\partial J}{\partial I}+(\delta-r)\frac{\partial J}{\partial z}-\delta J \leq 0 \text{ if } \frac{\partial J}{\partial z} = U_o(I),$$

with $U_o(I) = \phi^{1-\gamma}\mathbb{E}[\int_0^\infty e^{-\delta t}u(I_t)\,dt]$ with $I_0 = I$.

Duality

Following the convention in convex analysis, we define the convex conjugate $\tilde{u}(\cdot)$ of $u(\cdot)$ by

$$\tilde{u}(y) \equiv \max_{c} u(c) - yc.$$

We can show that the value function can be expressed as a minimum:

$$\begin{aligned} V(x) &= \min_{\lambda}, \min_{D_t} \widetilde{V}(x; \lambda, D) \\ &= \min_{\lambda}, \min_{D_t} \left\{ \mathbb{E}\left[\int_0^\infty e^{-\delta t} \left\{ (1 + D_t)\tilde{u}\left(\frac{\lambda e^{\delta t} H_t}{1 + D_t} \right) - D_t \phi^{1-\gamma} u(I_t) \right\} dt \right] \right. \\ &\qquad \left. + \lambda \mathbb{E}\left[\int_0^\infty H_t I_t dt \right] + \lambda x \right\}. \end{aligned}$$

A Related Problem: Non-tolerance for Decline in Living Standard

Consider the following problem (A joint work with Junkee Jeon and Yong Hyun Shin):

$$\max \mathbb{E}\left[\int_0^T e^{-\delta t} u(c_t) dt\right],$$

by choosing consumption and portfolio subject to $c_t \geq c_{t-}$.

This is an extreme form of habit formation in the spirit of Dusenberry's book, *Income, Saving and Theory of Consumer Behavior* (1949), Harvard University Press.

Non-tolerance for Decline in Living Standard

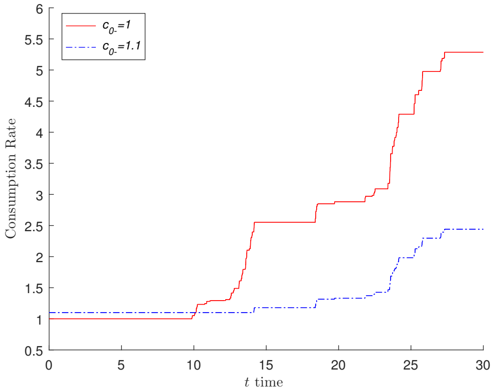
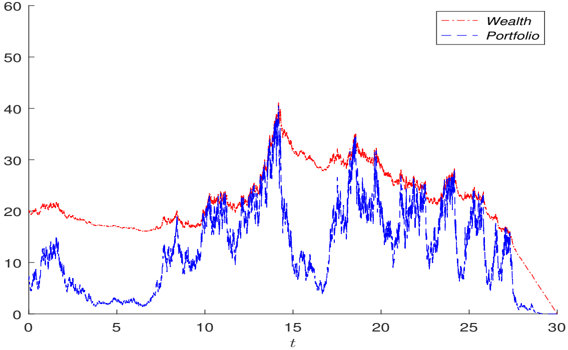


Figure: non-decreasing consumption process c,
$\delta = 0.07, r = 0.0371, \mu = 0.1123, \sigma = 0.1954, x_0 = 20,\ \gamma = 2.$

Non-tolerance for Decline in Living Standard



Simulation of wealth and portfolio with
$\rho = 0.07, r = 0.0371, \mu = 0.1123, \sigma = 0.1954, \gamma = 2$ and $x = 20$, $c_{0-} = 1$, $T = 30$.

Non-tolerance for Decline in Living Standard

We have the following result about the coefficient of relative risk aversion (ICRRA) implied by the optimal portfolio:

Theorem

(a) *If* $\liminf_{t\to T} ICRRA(t) = \limsup_{t\to T} ICRRA(t)$*, then* $ICRRA(t)$ *approaches to* 0 *as* $t \to T$*. And* $\lim_{T\to\infty} ICRRA(t) = \dfrac{1}{1-\lambda_-} < 1$*, where* λ_- *is the negavie root of a quadratic equation.*

(b) *If* $\liminf_{t\to T} \dfrac{dICRRA(t)}{dt} = \limsup_{t\to T} \dfrac{dICRRA(t)}{dt}$*, then there exists* $\delta > 0$ *such that* $ICRRA$ *decreases monotonically as time gets nearer to* T *for* $T - t \in (0, \delta)$*.*

That is, the agent acts as if he(she) is almost risk-neutral as the time horizon is short.

Non-tolerance for Decline in Living Standard

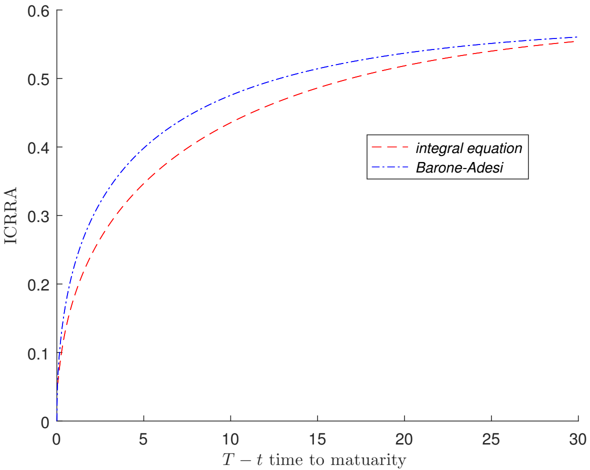


Figure: Implied coefficient of relative risk aversion with $\rho = 0.07, r = 0.0371, \mu = 0.1123, \sigma = 0.1954, \gamma = 2.$

Conclusion

- Under limited commitment there exists an interesting duality relationship between an optimal insurance contracting problem (arising from International Finance, Corporate Finance, Macroeconomics, and Political Economy) and an Optimal Consumption/Portfolio Selection Problem.
- We provide a bridge between the two strands of literature: the one on dynamic contracting with limited commitment and the other on optimal consumption/portfolio selection with dynamic constraints.