How Should Individuals Make a Retirement Plan in the Presence of Mortality Risks and Consumption Constraints?

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Section 1

Introduction

MOTIVATION

"Our approach to saving is all wrong: We need to think about monthly income, not net worth.."

Robert C. **Merton**
"The Crisis in Retirement Planning",
Harvard Business Review, 2014.

MOTIVATION

An approach by Merton (2014): A good framework in which to achieve **the goal** is to divide income needs into three categories:

- Category 1: Minimum guaranteed income
  - Income in this category must be inflation-protected and guaranteed for life, thus shielding the retiree from longevity risk, interest rate fluctuations, and inflation. Government benefits, such as Social Security, and any defined-benefit pensions would be included in this category.
- Category 2: Conservatively flexible income
  - Income in this category must be available in whole or in part to the individual at any time, for medical emergencies or other lump sum expenditures. And any assets remaining in the fund at her death would be available for bequests.
- Category 3: Desired additional income
  - Income for some individuals who may find that their anticipated total income and assets will not be enough to finance the level of retirement income they desire.

AIMS

This paper presents an integrated retirement planning,

- with a social security and a (DC-type) retirement pension,
- which does not only maximize the market value of the future income stream,
- but also guarantees a minimum income stream.

$\Rightarrow$ aims to give individuals a practical guide of retirement planning.

AIMS

This paper provides a life-cycle model in the presence of subsistence level in consumption,

- which is defined as a mode of consumption corresponding to the basic needs of life.
- Our basic needs of life can include somewhat welfare as well as a dietary needs, so the interpretation of our subsistence level can be correspondent to the 'weak' poverty line, as well as the 'strong' poverty line which is used to identify that part of population which is regard as absolutely poor

The subsistence level in consumption is closely related to the minimum guaranteed income.

- Maintaining a certain level of consumption can be achieved only by a guaranteed minimum income stream.

Main Findings

Our model shows

- that the movements of the optimal risky investments might dramatically change with the subsistence level in consumption,
- that the risky investment in the retirement pension can increase with the risk-free gross return and with risk aversion level when the risk-free rate and risk aversion level are both low, and
- that the risky investment in the retirement pension can decrease even when the market condition is favorable.

LITERATURE

Traditional retirement planning

- mainly focused on the market value of the future income stream rather than on the safety of the income stream (Blanchett and Ratner, 2015; Gomes and Michaelides, 2005; Horneff et al., 2008),
- simplified the peculiar structural characteristics of annuity products (Chen et al., 2006; Huang and Milevsky, 2008; Milevsky and Young, 2007).

LITERATURE

Optimal retirement planning in the presence of forced unemployment risks: Jang, Park, and Rhee (2013, JBF), Bensoussan, Jang, and Park (2015, OR), Jang and Park (2017, working)

- solve optimal consumption, portfolio choice, and retirement problem with forced unemployment risks such as health shocks and default of the firm.
- find optimal strategies for both complete(with unemployment insurance) and incomplete(without unemployment insurance) market cases.
- introduce a (ideal) fully-personalized unemployment insurance.
- find strong evidence that a small chance of forced unemployment can have a significant effect on the individual's optimal consumption, investment, and retirement behaviors.
- Comparing the complete market with the private unemployment insurance we show that the market innovation by the introduction of the private unemployment insurance might increase individuals' welfare, especially welfare of poor people and people with a low post-retirement leisure preference or who are placed in bad economic conditions.

LITERATURE

An ideal private unemployment insurance by Jang, Park, and Rhee (2013, JBF)

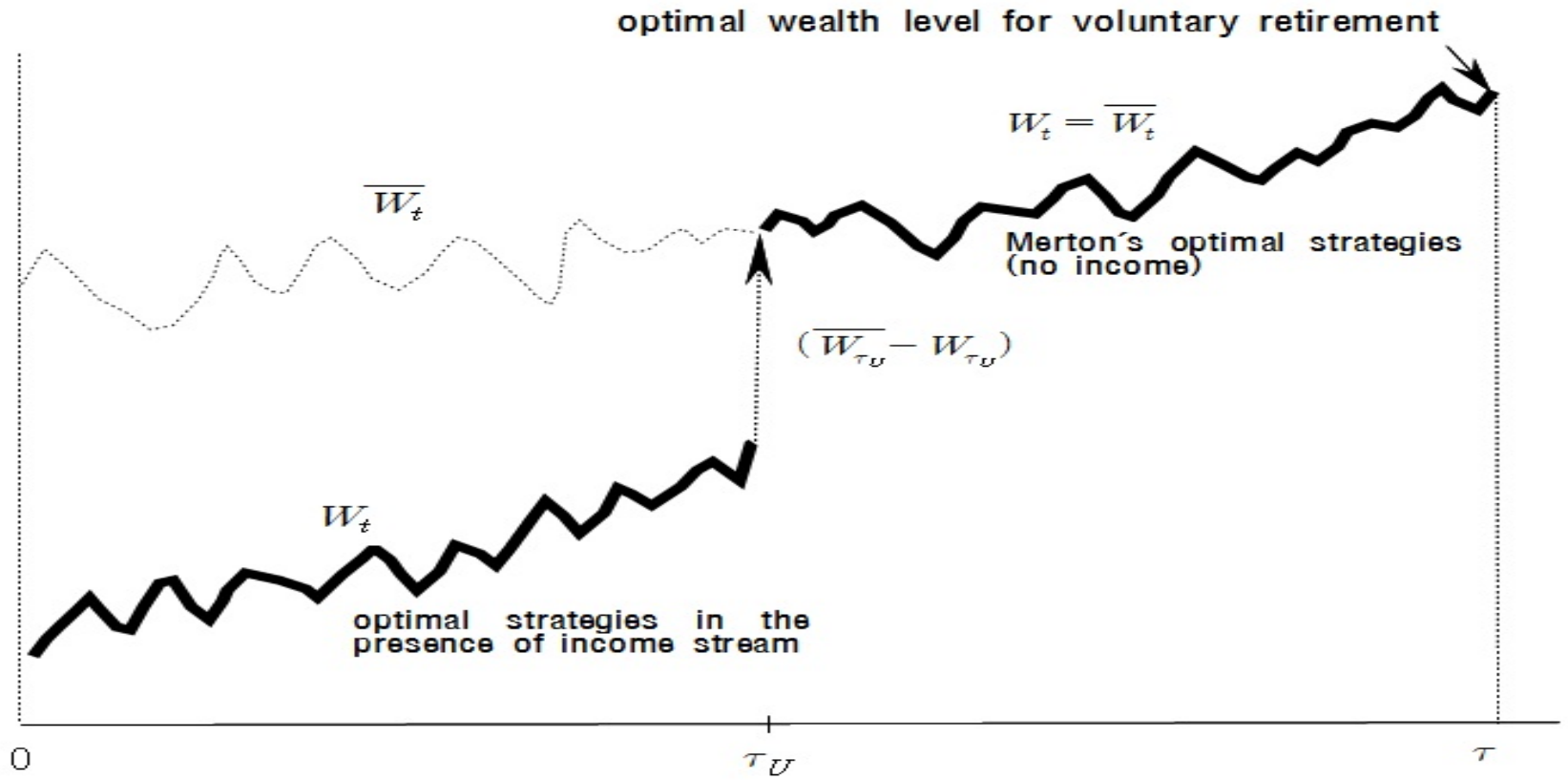


Figure: The jump size should be the coverage of the unemployment insurance.

LITERATURE

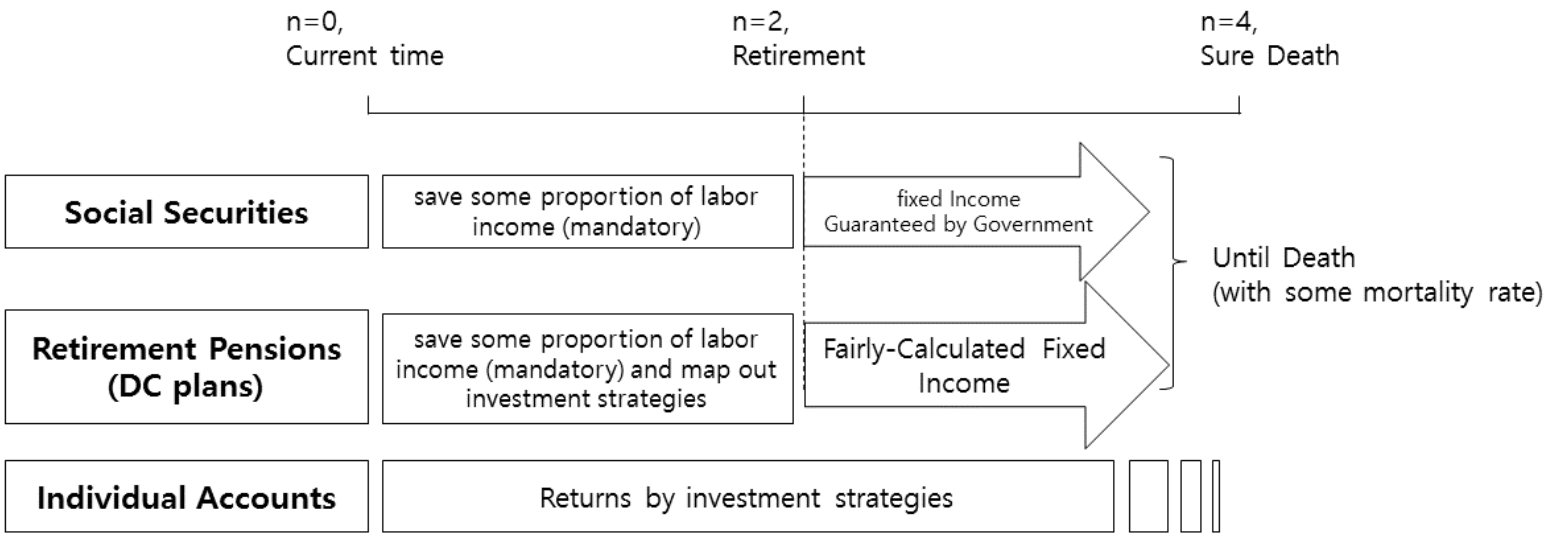
Optimal consumption and investment strategies for the retired in the presence of longevity risk: Jang, Koo, and Rhee (2016, ET)

- study asset demands and consumption of an individual at the end of her life cycle.
- present an ideal market where complete insurance against longevity risk is available: the market consists of original assets, e.g., stocks and bonds, and annuities and life-insurance contracts linked to these assets. We also study asset demand and consumption for the other two cases: the no insurance case where there is no insurance available against longevity risk, and the partial insurance case where only bond-linked annuities exist for longevity insurance.
- find the proportion of wealth invested in risky assets to total savings in the complete market case is similar to the proportion in the no insurance case. It is, however, in general larger in the complete market case than in the partial insurance case. Thus balanced insurance market development might be important.
- find that innovations in the insurance industry which lower costs of providing personalized insurance may result in a significant shift in asset demands, particularly in the demand for risky assets.

Section 2

Model

THREE INVESTMENT VEHICLES



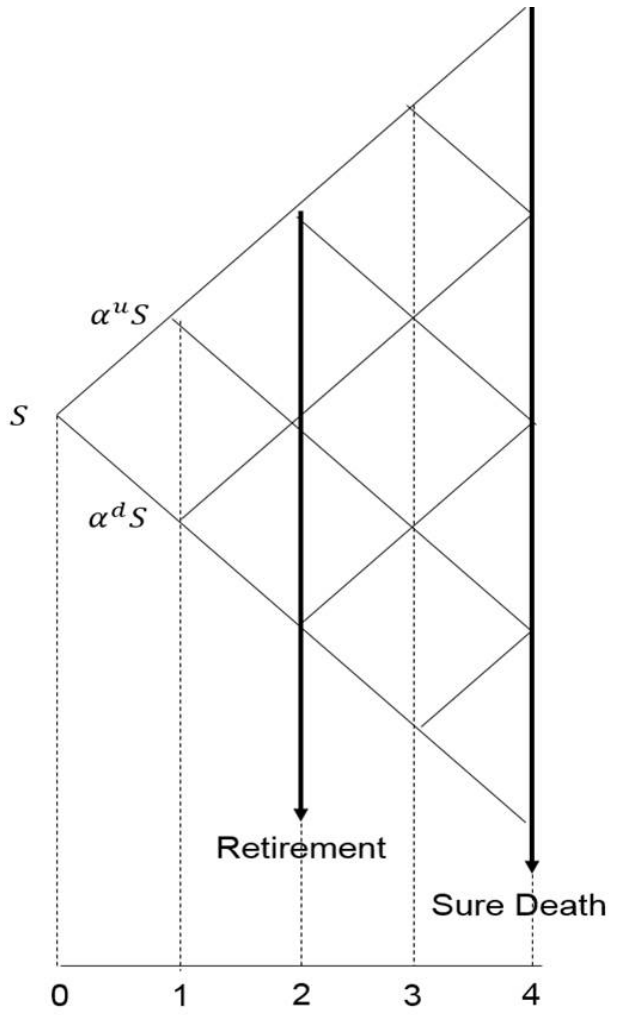
Model Description

Individual's Life-time Events

- A four-period binomial tree model.
- The current time 0.
- Fixed retirement at time 2.
  - Before the retirement, get labor income, save some portion of labor income in a social security and a DC pension.
  - After the retirement, get income from a social security and a DC pension.
- Longevity risk modelled by a mortality rate δ_n at time $n = 1, 2, 3, 4$.
- Sure Death at time 4, $\delta_4 = 1.0$.

Financial Market: A Binomial Tree Model

- a risk-free asset (a bond) with a constant gross return R,
- a risky asset (a stock) with a stochastic gross return α (α^u with probability p^u, α^d with probability $p^d = 1 - p^u$).



PRE-RETIREMENT WEALTH PROCESSES

Before retirement,

- Individuals should save a portion of their labor income with a given rates θ_n^S and θ_n^{RP} into the social security and the retirement pension, respectively.
- The individuals choose the risky investment rate π_n^{RP} $(n = 0, 1)$ in the retirement pension.
- They also choose the consumption level C_n and the risky investment rate π_n.

$$\begin{cases} W_n = (R + (\alpha_n - R)\pi_{n-1})(W_{n-1} - C_{n-1} + (1 - \theta_{n-1}^S - \theta_{n-1}^{RP})I_{n-1}), \\ W_n^{RP} = (R + (\alpha_n - R)\pi_{n-1}^{RP})(W_{n-1}^{RP} + \theta_{n-1}^{RP} I_{n-1}), \end{cases} \quad \text{for } n = 1, 2.$$

POST-RETIREMENT WEALTH PROCESS

After retirement,

- The social security income is assumed to be proportional to the last income I_1 with a given constant ratio ρ^S.
- The regular payment $I^{RP} \equiv \rho^{RP} W_2^{RP}$ of the retirement pension is fairly determined:

$$W_2^{RP} = I^{RP} + (1 - \delta_3) I^{RP}/R \;\Rightarrow\; \rho^{RP} = \frac{1}{1 + (1 - \delta_3)/R}.$$

- Individuals still choose the consumption level C_n and the risky investment rate π_n.

$$W_n = (R + (\alpha_n - R)\pi_{n-1})(W_{n-1} - C_{n-1} + \rho^S I_1 + \rho^{RP} W_2^{RP}), \qquad \text{for } n = 3, 4.$$

Consumption and Investment Constraints

Consumption Constraints from Merton's Approach:

- $\underline{C}^b$ ($\underline{C}^a$) : the subsistence level in consumption before (after) the retirement. Can be considered as a Minimum Guaranteed Income.
- This constraint the possibilities to substitute consumption intertemporally.

$$C_n \geq \begin{cases} \underline{C}^b & \text{for } n = 0, 1, \\ \underline{C}^a & \text{for } n = 2, 3, \end{cases}$$

Investment Constraints in DC pension Management:

- $\overline{\pi}$: the upper position limit,
- $\underline{\pi}$: the lower position limit,
- In order to prevent an illegal trading through a large-size retirement pension, a few countries adopt position limits in the investment rate in retirement pensions.

$$\underline{\pi} \leq \pi_n^{RP} \leq \overline{\pi} \quad \text{for } n = 0, 1.$$

CONTINUATION VALUE FUNCTION: EPSTEIN-ZIN UTILITY

The value function where the individual is alive (dead) at time n, ${V_n}^L$ (${V_n}^D$, resp.) is defined as

$$V_n^L = \max_{c_n, \pi_n, \pi_n^{RP}} \left(C_n^{1-\rho} + \beta \left(\delta_{n+1} \mathbb{E}_n \left[{V_{n+1}^D}^{1-\gamma} \right] + (1-\delta_{n+1}) \mathbb{E}_n \left[{V_{n+1}^L}^{1-\gamma} \right] \right)^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}, n = 0, 1,$$

$$V_n^L = \max_{c_n, \pi_n} \left(C_n^{1-\rho} + \beta \left(\delta_{n+1} \mathbb{E}_n \left[{V_{n+1}^D}^{1-\gamma} \right] + (1-\delta_{n+1}) \mathbb{E}_n \left[{V_{n+1}^L}^{1-\gamma} \right] \right)^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}, \quad n = 2, 3,$$

$$V_n^D = W_n, \qquad n = 1, 2, 3, 4,$$

- β : the subjective discount rate,
- γ : the relative risk aversion (RRA),
- $\eta \equiv \rho^{-1}$: the elasticity of intertemporal substitution (EIS) in consumption,

THE PROBLEM

To find

$$V_n^L = \max_{c_n, \pi_n, \pi_n^{RP}} \left(C_n^{1-\rho} + \beta \left(\delta_{n+1} \mathbb{E}_n \left[{V_{n+1}^D}^{1-\gamma} \right] + (1-\delta_{n+1}) \mathbb{E}_n \left[{V_{n+1}^L}^{1-\gamma} \right] \right)^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}, \; n = 0, 1,$$

$$V_n^L = \max_{c_n, \pi_n} \left(C_n^{1-\rho} + \beta \left(\delta_{n+1} \mathbb{E}_n \left[{V_{n+1}^D}^{1-\gamma} \right] + (1-\delta_{n+1}) \mathbb{E}_n \left[{V_{n+1}^L}^{1-\gamma} \right] \right)^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}, \quad n = 2, 3,$$

$$V_n^D = W_n, \qquad n = 1, 2, 3, 4,$$

subject to

$$\begin{cases} W_n = (R + (\alpha_n - R)\pi_{n-1})(W_{n-1} - C_{n-1} + (1 - \theta^S - \theta^{RP})I_{n-1}), \\ W_n^{RP} = (R + (\alpha_n - R)\pi_{n-1}^{RP})(W_{n-1}^{RP} + \theta^{RP} I_{n-1}), \end{cases} \quad \text{for } n = 1, 2,$$

$$W_n = (R + (\alpha_n - R)\pi_{n-1})(W_{n-1} - C_{n-1} + \rho^S I_2 + \rho^{RP} W_2^{RP}), \quad \text{for } n = 3, 4,$$

and

$$C_n \geq \begin{cases} \underline{C}^b & \text{for } n = 0, 1, \\ \underline{C}^a & \text{for } n = 2, 3, \end{cases} \qquad \text{and} \qquad \underline{\pi} \leq \pi_n^{RP} \leq \overline{\pi} \quad \text{for } n = 0, 1.$$

Section 3

Data and Parameter Estimation

BASELINE PARAMETERS

Parameter	Baseline	Source
time interval (Δt, year)	15	-
15-year risk-free gross return (R)	1.33	Ibottson (30-day T-bill)
probability of up markets (p^u)	0.722719	Yahoo Finance (S&P 500)
15-years stock gross return in up markets (α^u)	4.28534	Yahoo Finance (S&P 500)
- in down markets (α^d)	0.409691	Yahoo Finance (S&P 500)
current wealth (W)	\$347200	SCF (group aged 35-44)
labor income ($\{I_0, I_1\}$)	$\{\$600434, \$609908\}$	BLS(Current Population Survey, 2013 1Q)
portion of income saved in SS ($\{\theta_1^S, \theta_2^S\}$)	$\{7.19\%, 10.94\%\}$	3% at 25, yearly increase of 0.25%
output-input ratio of SS (ρ^S)	43%	AARP
portion of income saved in RP (θ^{RP})	10%	401k recommendation
upper position limit ($\overline{\pi}$)	1.0	401k
lower position limit ($\underline{\pi}$)	0.0	401k
mortality rate ($\{\delta_1, \delta_2, \delta_3\}$)	$\{4.0\%, 13.0\%, 38.1\%\}$	SSPL Table
subjective discount rate (β)	0.86	99% per yr
coefficient of relative risk aversion (γ)	5	-
coefficient of EIS (η)	1/3	Vissing-Jorgensen (2002)

Section 4

Implication

IMPORTANCE OF SUBSISTENCE LEVEL

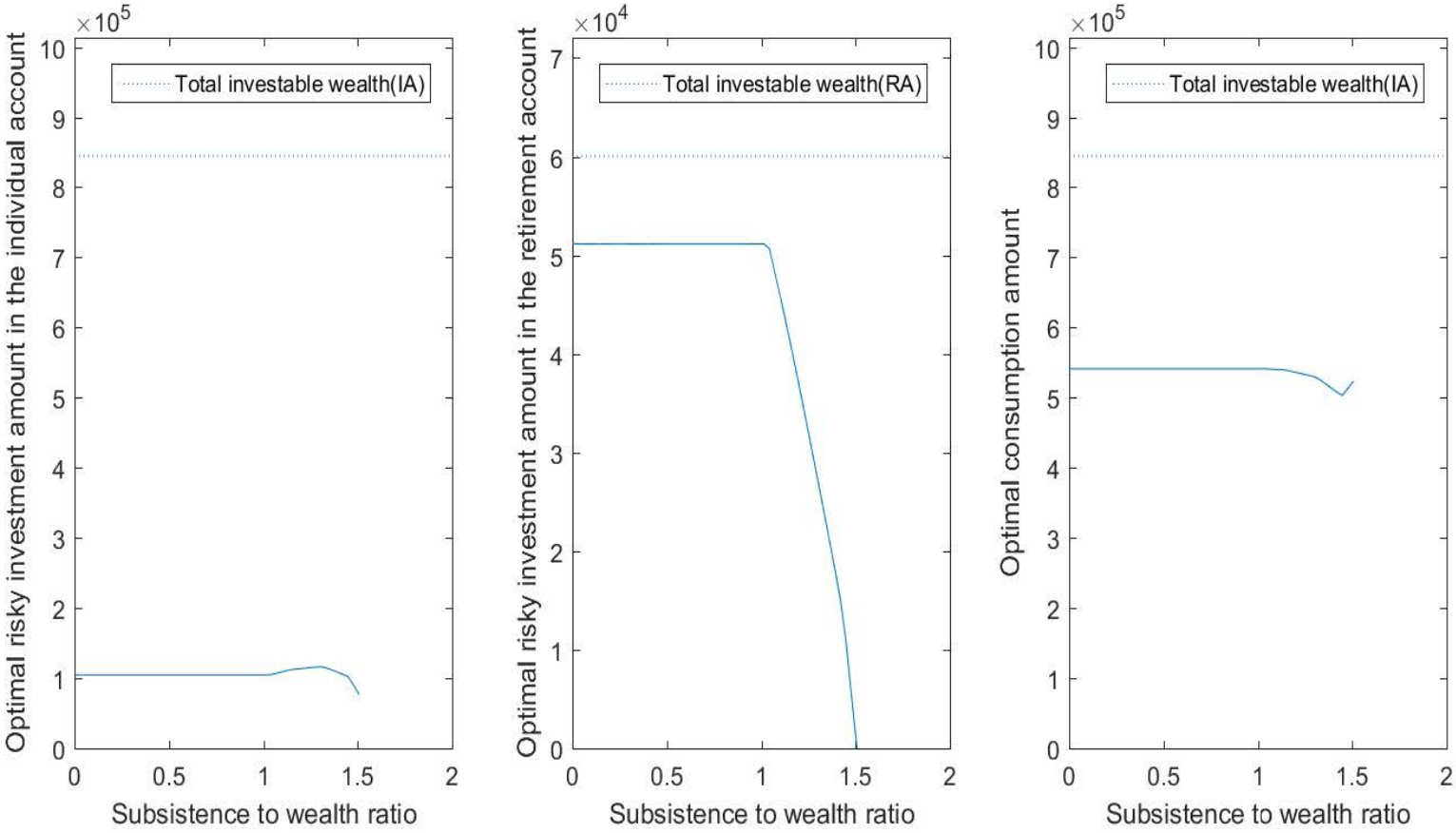


Figure: The optimal risky investment in the individual account (left), in the retirement account (middle), and the optimal consumption amount (right) as functions of the subsistence level in the consumption-to-wealth ratio ($\underline{C}^a/W = \underline{C}^b/W$).

*The median income-to-wealth ratio of the group with income from the bottom 20% to the bottom 40% is 1.13 and that of the group with income from the bottom 40% to the bottom 60% is 1.85.

RISK-FREE RATE WITH SUBSISTENCE LEVEL 1.1

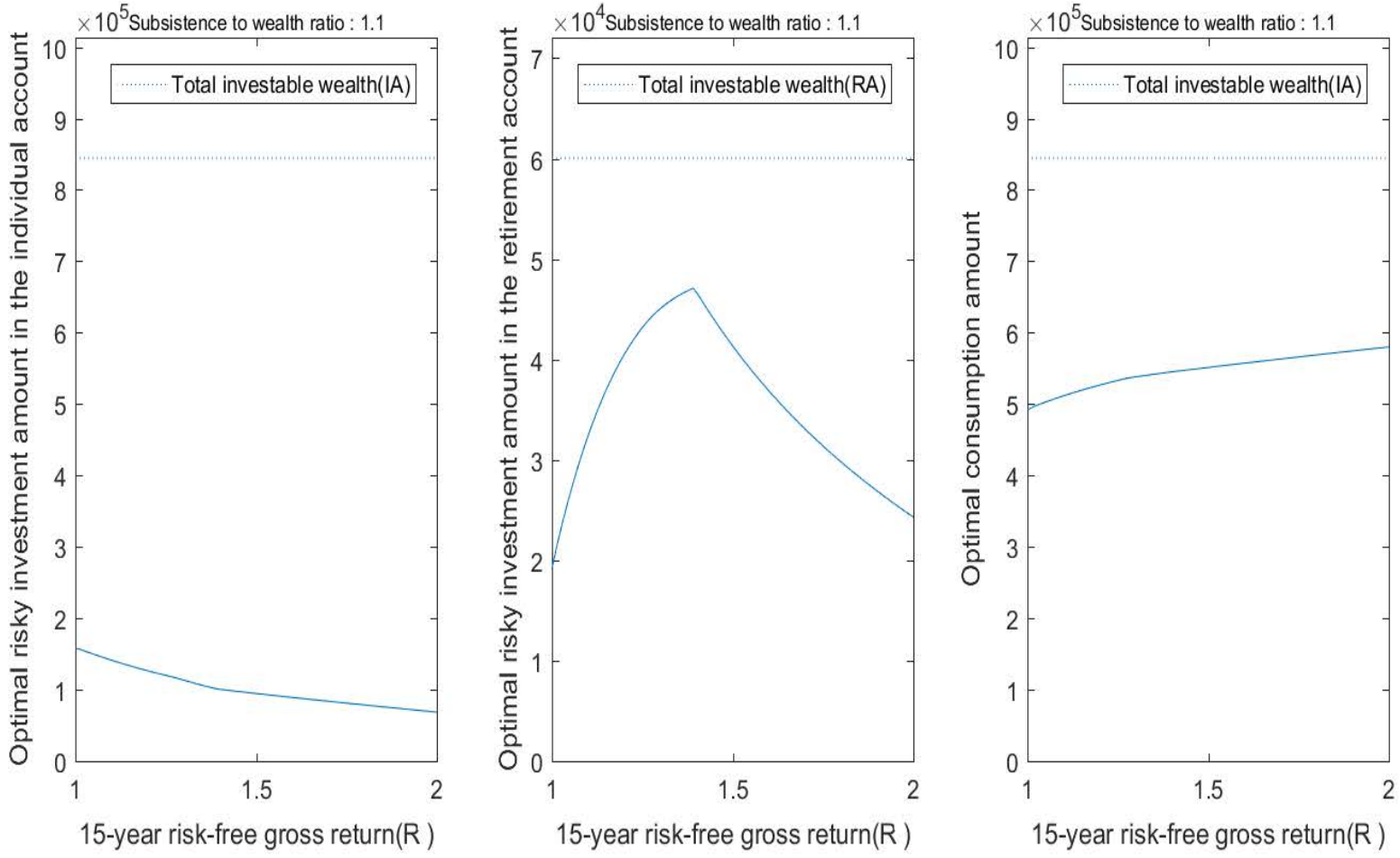


Figure: The optimal risky investment in the individual account (left), in the retirement account (middle), and the optimal consumption amount (right) as functions of 15-years risk-free gross return R.

UP-MKT. PROB. WITH SUBSISTENCE LEVEL 1.1

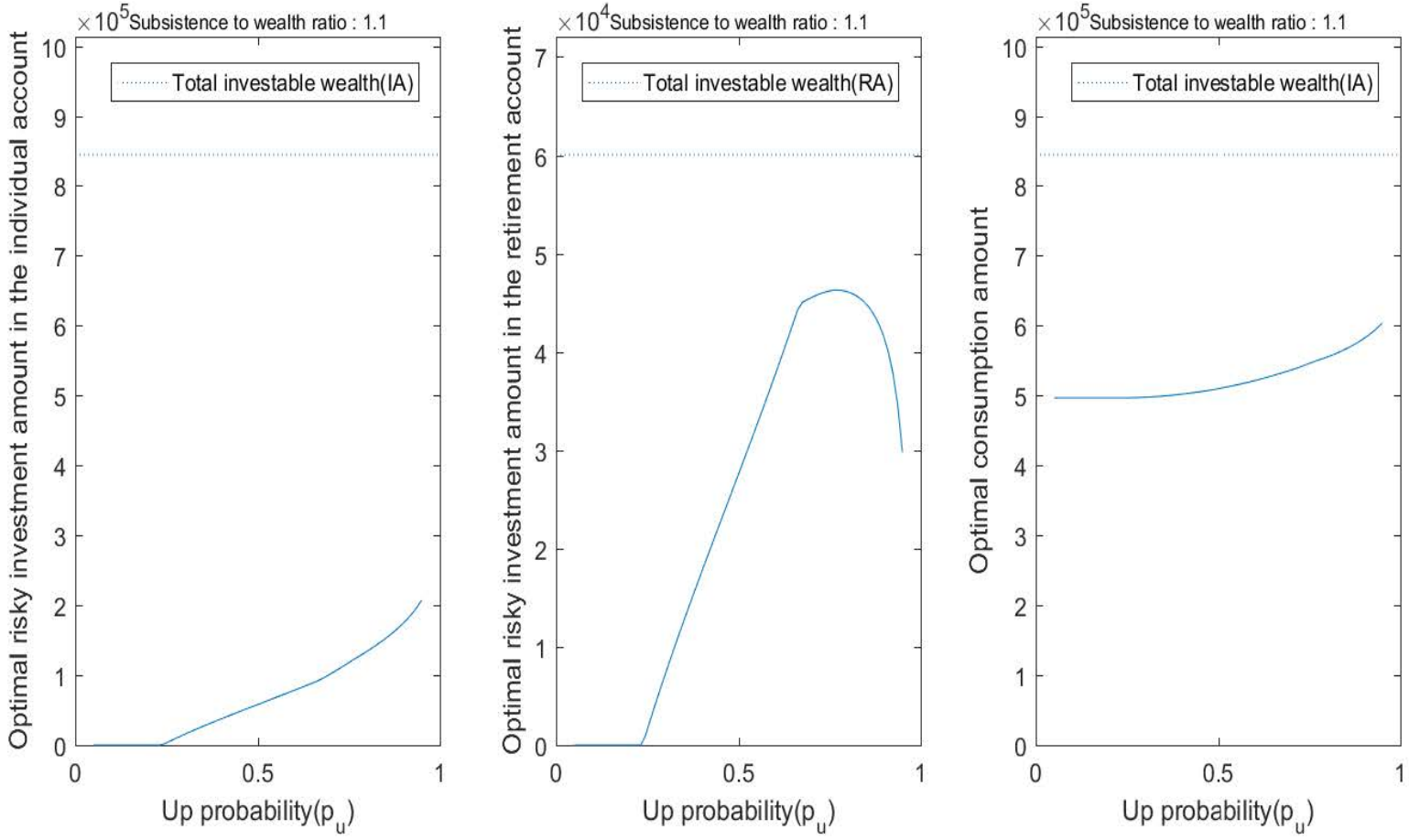


Figure: The optimal risky investment in the individual account (left), in the retirement account (middle), and the optimal consumption amount (right) as functions of up probability p^u.

RRA WITH SUBSISTENCE LEVEL 1.1

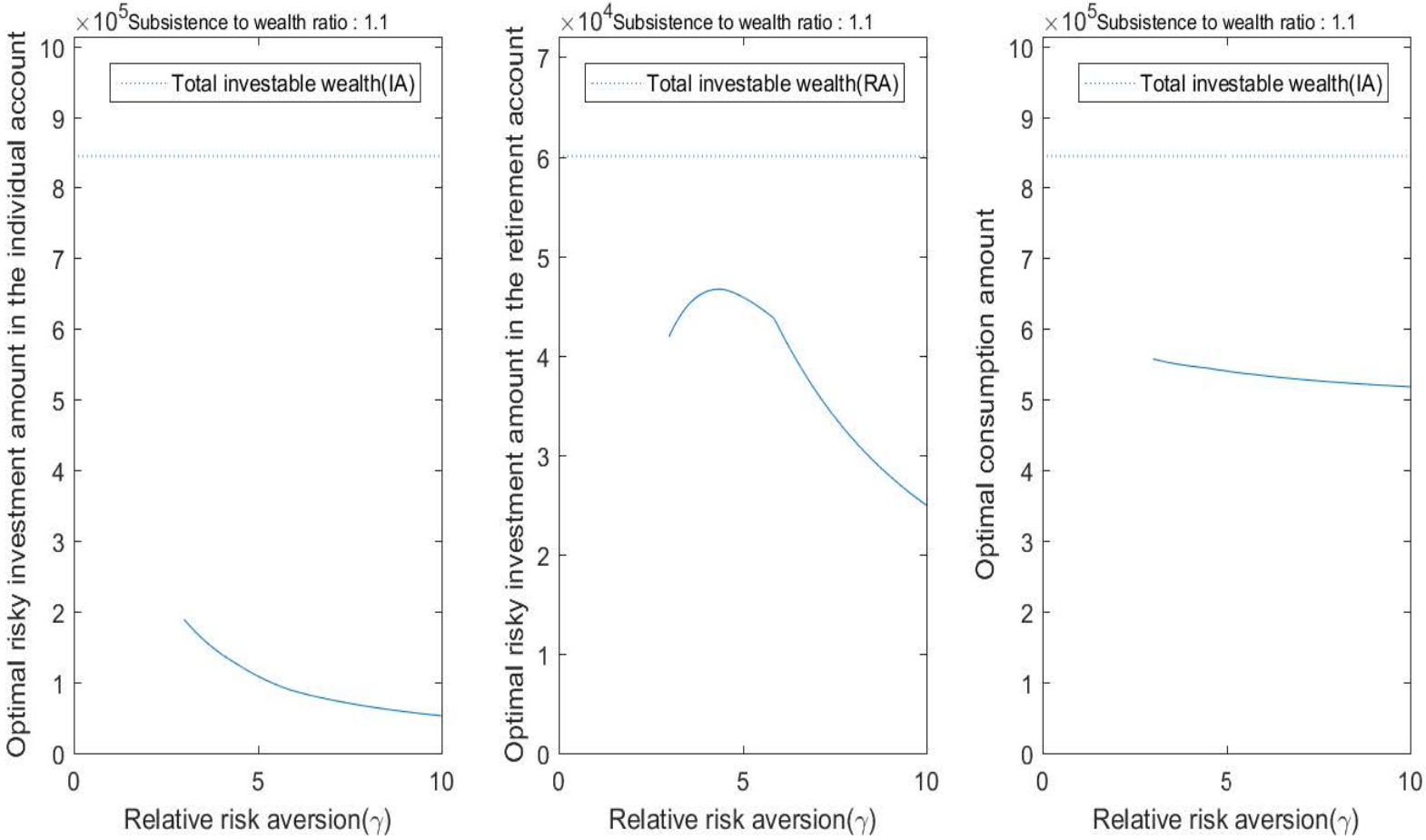


Figure: The optimal risky investment in the individual account (left), in the retirement account (middle), and the optimal consumption amount (right) as functions of the relative risk aversion (RRA) level γ.

EIS WITH SUBSISTENCE LEVEL 1.1

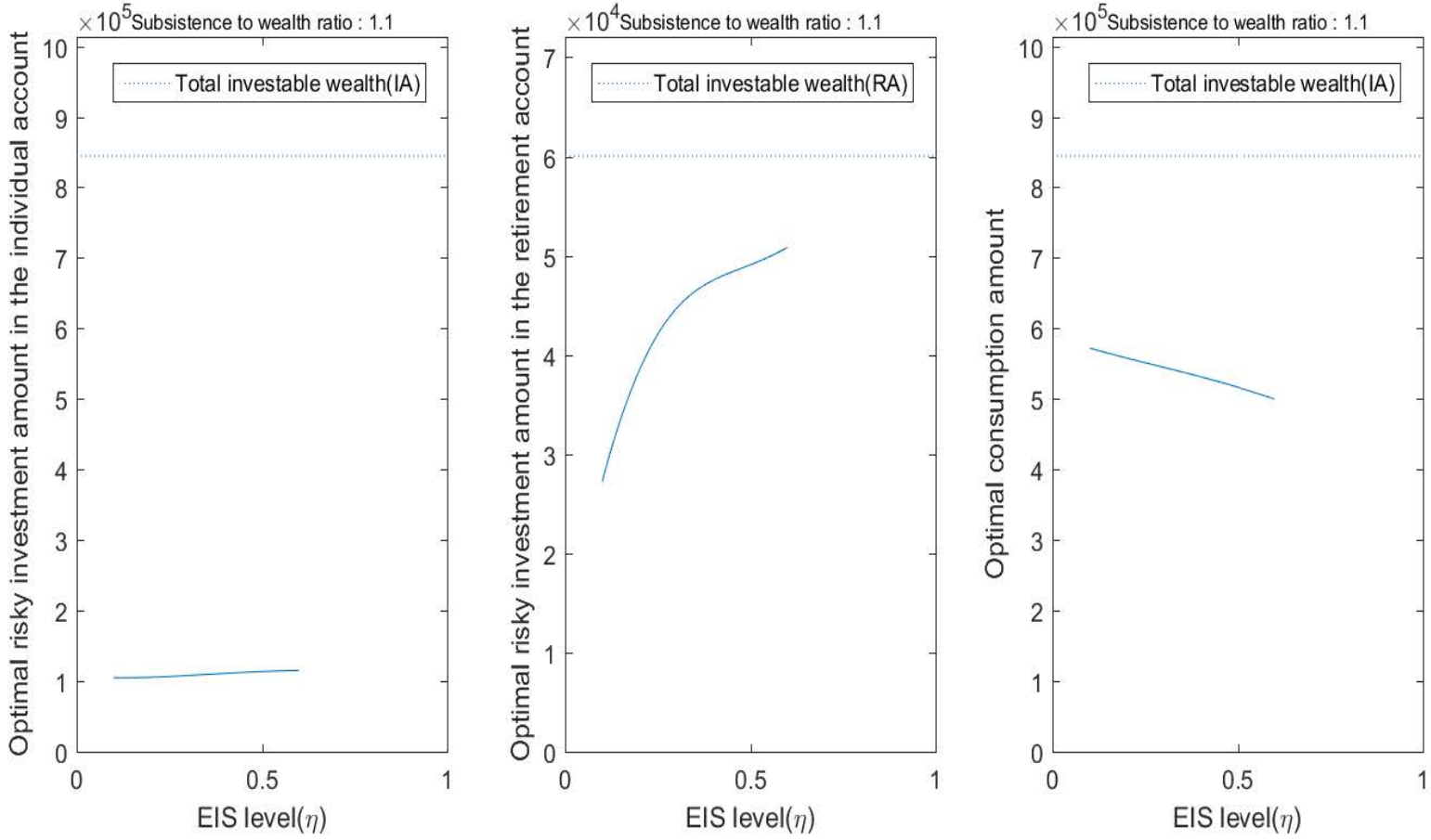


Figure: The optimal risky investment in the individual account (left), in the retirement account (middle), and the optimal consumption amount (right) as functions of the elasticity of intertemporal substitution (EIS) on consumption.

Section 5

Conclusions

SUMMARY

- This paper presents an integrated retirement planning,
  - with a social security and a (DC-type) retirement pension,
  - which does not only maximize the market value of the future income stream,
  - but also guarantees a minimum income stream.
- Our model shows
  - that the movements of the optimal risky investments might dramatically change with the subsistence level in consumption,
  - that the risky investment in the retirement pension can increase with the risk-free gross return and with risk aversion level when the risk-free rate and risk aversion level are both low, and
  - that the risky investment in the retirement pension can decrease even when the market condition is favorable.

EXTENSIONS

We can consider

- optimal voluntary retirement (instead of a fixed retirement date),
- inflation and interest rate fluctuation,
- random shocks of unemployment,
- tax benefits in the retirement pension,
- options to receive benefits of the social security and/or the retirement pension at once, and
- others.