Pricing Options Using Machine Learning Methods

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- Financial Markets follow a complex pattern and are characterized by stochastic behavior resulting in multivariate and highly nonlinear option pricing functions.
- Modern parametric option pricing models establish a stationary nonlinear relationship between theoretical option prices and its variables.
- Parametric models may fail to adjust to such rapidly changing behavior in derivative markets.
- In addition to this, market participants always need more accurate models or methods that can be utilized in real-world applications.
- Therefore, efforts are being made to develop machine learning methods that may overcome the limitations of parametric models.

- Machine learning methods have attracted a lot of attention by researchers and practitioners the last few decades.
- These methods typically includes highly data-intensive model-free approaches that complement traditional parametric methods.
- One characteristic of such methods is their independence of the assumptions of continuous finance theory.
- The strength of these methods is that no complex models are presumed from which prices are deducted. Actually, the structure of the problem is induced from the data.

In this study, we utilize following two prominent machine learning methods to model option prices :

1. Dynamic Neural Networks (DNN)
2. Support Vector Regression (SVR)

The above methods are applid on following data set to model and forecast option prices :
Input Matrix : $[S, X, T, \sigma, r]$ of order 13645×5.
Output Matrix : [C] of order 13645×1.

- In dynamic networks, the output depends not only on the current input to the network, but also on the current or previous inputs, outputs or states of the network. It has recurrent (feedback) connections, which means that the current output is a function of outputs at previous times also.
- Since financial market posses memory which can be observed directly from data, we use dynamic neural network to model and predict option prices.
- Dynamic neural networks are good at time series prediction. We design a network which can predict option prices by taking its independent variables i.e. strike price (X), spot price (S), time to maturity (T), volatility (σ) and interest rate (r) and dependent variable call option prices(C).

A neural network model implementation can be divided into the following three parts:

1. Training : In this part, 70% of data is present during training and the network is adjusted according to its error.
2. Validation : In this part, 15% of data is used to measure network generalization and to halt when generalization stops improving.
3. Testing : In this part, 15% of data is used. These have no effect on training and so provide an independent measure of network performance during and after training.

We build following two dynamic neural network models for option price prediction:

1. Time Delay Neural Networks (TDNN)
2. Nonlinear Autoregressive with External Inputs (NARX)

- The TDNN model also involves two different set of time series $X(t)$ and $Y(t)$. We predict future values of a set time series $Y(t)$ from past values of a second set of time series $X(t)$.

$$Y(t) = f(X(t-1), X(t-2), \cdots X(t-d))$$

- TDNN is the simplest dynamic neural network which consists of a feed-forward neural network with a tapped delay line at input.
- This is well suited for multi-dimensional time series prediction.

- In this study, the TDNN network is a two-layer feedforward network with a sigmoid transfer function in the hidden layer and a linear transfer function in the output layer.
- We define our time series problem and arrange a set of time series input vectors as column vectors which include five columns for the input data set and arrange a set of output variable data which includes one column.
- Our network architecture is, thus, five (inputs)-two (hidden layer)- one (output).(Bases on error correction rule).
- We use Levenberg-Marquardt Algorithm to train data which is the most commonly-used algorithm for such data and architecture.

- In NARX model, once again, two different set of time series $X(t)$ and $Y(t)$ are involved. We predict the future values of a time series $Y(t)$ from past values of that time series and past values of a second time series $X(t)$.
- Nonlinear autoregressive network with exogenous inputs (NARX) is a recurrent dynamic network, with feedback connections enclosing several layers of the network. The NARX model can be expressed mathematically thus,

$$\begin{aligned} Y(t) &= f(Y(t-1), Y(t-2), \cdots Y(t-d), \\ &\quad X(t-1), X(t-2), \cdots X(t-d)). \end{aligned}$$

  where the next value of the dependent output signal Y(t) is regressed on previous values of the output signal and previous values of an independent

- We treat here the derivative market as a dynamical system which changes on continuous feedback and use the NARX Model to model such a system. The input matrix for the NARX network is $V = [X, S, T, \sigma, r]$ which is again chosen on the basis of the BSM model. The output variable for the network is $W = [C]$.
- The network architecture of NARX is similar to TDNN with feedback loop.
- We use the Levenberg-Marquardt Algorithm which is the most commonly-used algorithm for such data and architecture.

- We develop a nonlinear SVM regression model by using a nonlinear kernel function $K(X_i, X_j) = \phi(X_i)^T \phi(X_j)$, where $\phi(X_i)$ is a transformation that maps X_i to a high-dimensional space.
- The idea behind SVR is to estimate coefficient values w and b that optimize the generalization ability of the regressors by minimizing the following regularized loss function :

$$\min_{w,b} \frac{1}{2} w^T w + C \sum_{j=1}^{P} L_\epsilon(Y_j, f(X_j))$$

where Y (C) denotes the target function observed in market data, $X_i = [X, S, T, \sigma, r]$ is input data matrix and P (13645) denotes the number of data points considered.

- In addition, $f(X)$ is an SVR function approximation and is given by :

$$f(X) = w^T \phi(X) + b$$

and $L_\epsilon(Y_j, f(X_j))$ is called the Vapnik's ϵ-loss function.

- In the above formulations $\phi(X) : R^N \rightarrow R^{N_h}$ represents a nonlinear transformations of the input space to high-dimensional feature space which can be infinite-dimensional.
- The constant $C(> 0)$ determines the trade-off between amount up to which deviations larger than ϵ are tolerated and the flatness of the estimated model.

- To successfully apply the methodology for nonlinear regression problems it is necessary to apply the kernal trick by choosing a proper kernal function :

$$K(X_j, X_i) = \phi(X_j)^T \phi(X_i)$$

A function that is symmetric, continuous and that satisfies Mercer's condition is admissible for our case.

- We use the Gaussian Radial Basis Function (RBF) kernal which is a widespread kernal function that is admissible for use with SVR and is given by

$$K(X_j, X_i) = \exp(-\gamma \|X_j - X_i\|^2)$$

- The RBF are general purpose kernals used when there is noprior knowledge about the data.

- We use tuning function to find the optimum parameter values using grid search methods. On the basis of RMSE, we find values of hyper-parameters C = 10000 and $\gamma = 0.006$.
- We train the data with standard and formula interface on training set data. The method converges at RMSE = 332.13. Here, we fixed $\epsilon = 0.1$ and 7548 support vectors were utilized to reach the optimum solution.
- Further, we predict option prices on different moneyness criteria using SVR.

The predicted option prices are given in following Table:

Table: Theoretical option prices

	K=3800	K=3900	K=4000	K=4100	K=4200
BSM	318.18	257.23	205.55	159.81	122.71
NN(NARX)	351.56	273.72	209.11	157.29	115.748
NN(TDNN)	352.31	290.94	233.09	180.11	133.35
SVR	329.88	271.81	217.09	165.89	118.34

We use Diebold-Marino test to compare forecast and obtain following results :

- The NARX model outperforms the TDNN model marginally, not inferentially. That is to say, the inclusion of an extra information input has a marginal impact on forecasts of option prices.
- The TDNN model outperforms the SVM model inferentially to forecast option prices.
- The NARX model outperforms the SVM model inferentially to forecast option prices.
- Thus,both machine-learning methods,Artificial Neural Networks(ANN) and Support Vector Machines perform similarly in the forecast of option prices. But, ANN models outperforms SVM model to forecast option prices in Indian market.

- Pricing options is a challenging problem because financial time series of option prices and their variables are greatly influenced by international, economic and political events which might not be captured by parametric models. Therefore, machine learning methods are better alternatives to price these securities.
- But, we require parametric models also. Hence, if machine learning methods predict any future risk situations, these situations can be managed by changing the policy factors of parametric models.

THANK YOU !